I realize that, no matter how careful I have endeavored to be, occasional errors may still exist. I should be grateful if you would be k[illegible] enough to notify me as you discover them either in the book or in this manual.

Sincerely,

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Solutions to problems of Cheng's Book

(For the use of instructors only.)

Capes Dominoslan

Chapter 2

P.2-1 (a) $\bar{a}_A = \frac{\bar{A}}{A} = \frac{\bar{a}_x + \bar{a}_y 2 - \bar{a}_z 1}{\sqrt{1^2 + 2^2 + (-3)^2}} = \frac{1}{\sqrt{14}}(\bar{a}_x + \bar{a}_y 2 - \bar{a}_z 3)$

(b) $|\bar{A} - \bar{B}| = |\bar{a}_x + \bar{a}_y 6 - \bar{a}_z 4| = \sqrt{1^2 + 6^2 + 4^2} = \sqrt{53}$

(c) $\bar{A} \cdot \bar{B} = 0 + 2(-4) + (-3) = -11$

(d) $\theta_{AB} = \cos^{-1}(\bar{A} \cdot \bar{B}/AB) = \cos^{-1}(-11/\sqrt{14}\sqrt{11}) = 135.5°$

(e) $\bar{A} \cdot \bar{a}_C = \bar{A} \cdot \frac{\bar{C}}{C} = \bar{A} \cdot \frac{1}{\sqrt{29}}(\bar{a}_x 5 - \bar{a}_z 2) = \frac{11}{\sqrt{29}}$

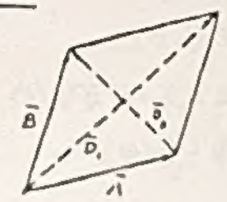
(f) $\bar{A} \times \bar{C} = -\bar{a}_x 4 - \bar{a}_y 13 - \bar{a}_z 10$

(g) $\bar{A} \cdot (\bar{B} \times \bar{C}) = (\bar{A} \times \bar{B}) \cdot \bar{C} = -42$

(h) $(\bar{A} \times \bar{B}) \times \bar{C} = \bar{B}(\bar{A} \cdot \bar{C}) - \bar{A}(\bar{C} \cdot \bar{B}) = \bar{a}_x 2 - \bar{a}_y 40 + \bar{a}_z 5$
$\bar{A} \times (\bar{B} \times \bar{C}) = \bar{B}(\bar{A} \cdot \bar{C}) - \bar{C}(\bar{A} \cdot \bar{B}) = \bar{a}_x 33 + \bar{a}_y 44 - \bar{a}_z 11$

P.2-2 Position vectors of the three corners:
$\overrightarrow{OP_1} = \bar{a}_y - \bar{a}_z 2$, $\overrightarrow{OP_2} = \bar{a}_x 4 - \bar{a}_y - \bar{a}_z 3$, $\overrightarrow{OP_3} = \bar{a}_x 6 + \bar{a}_y 3 + \bar{a}_z 5$
Vectors representing the three sides of the triangle:
$\overrightarrow{P_1P_2} = \overrightarrow{OP_2} - \overrightarrow{OP_1} = \bar{a}_x 4 - \bar{a}_z 1$, $\overrightarrow{P_2P_3} = \bar{a}_x 2 + \bar{a}_y 4 + \bar{a}_z 8$, $\overrightarrow{P_3P_1} = -\bar{a}_x 6 - \bar{a}_y - \bar{a}_z 7$
(a) $\overrightarrow{P_1P_2} \cdot \overrightarrow{P_2P_3} = 0$. $\therefore \Delta P_1P_2P_3$ is a right triangle.
(b) Area of triangle $= \frac{1}{2}|\overrightarrow{P_1P_2} \times \overrightarrow{P_2P_3}| = 17.1$

P.2-3

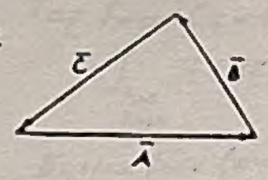


$\bar{D}_1 = \bar{B} + \bar{A}$, $\bar{D}_2 = \bar{B} - \bar{A}$
$\bar{D}_1 \cdot \bar{D}_2 = (\bar{B} + \bar{A}) \cdot (\bar{B} - \bar{A})$
$= \bar{B} \cdot \bar{B} - \bar{A} \cdot \bar{A} = 0$
$\therefore \bar{D}_1 \perp \bar{D}_2$

P.2-4 From $\bar{A} \cdot \bar{B} = \bar{A} \cdot \bar{C}$, we have $\bar{A} \cdot (\bar{B} - \bar{C}) = 0$. (1)
From $\bar{A} \times \bar{B} = \bar{A} \times \bar{C}$, we have $\bar{A} \times (\bar{B} - \bar{C}) = 0$. (2)
(1) implies $\bar{A} \perp (\bar{B} - \bar{C})$ and (2) implies $\bar{A} \parallel (\bar{B} - \bar{C})$. Since $\bar{A}$ is not a null vector, (1) and (2) cannot hold at the same time unless $(\bar{B} - \bar{C})$ is a null vector. Thus, $\bar{B} - \bar{C} = 0$ or $\bar{B} = \bar{C}$.

P.2-5 $\bar{a}_A \cdot \bar{a}_B = \cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$.

P 2-6



$\bar{A} + \bar{B} + \bar{C} = 0$

$\bar{A} \times$: $\bar{A} \times \bar{B} = \bar{C} \times \bar{A}$

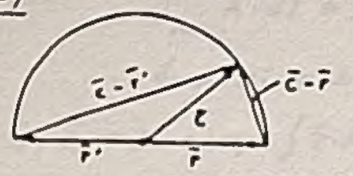
$\bar{C} \times$: $\bar{C} \times \bar{A} = \bar{B} \times \bar{C}$

$\bar{B} \times$: $\bar{B} \times \bar{C} = \bar{A} \times \bar{B}$

Magnitude relations: $AB \sin\theta_{AB} = CA \sin\theta_{CA} = BC \sin\theta_{BC}$

Hence $\dfrac{A}{\sin\theta_{BC}} = \dfrac{B}{\sin\theta_{CA}} = \dfrac{C}{\sin\theta_{AB}}$.

P 2-7



$\bar{r}' = -\bar{r}$, $r' = r$

$(\bar{c} - \bar{r}') \cdot (\bar{c} - \bar{r}) = (\bar{c} + \bar{r}) \cdot (\bar{c} - \bar{r})$

$= 0.$

$\therefore (\bar{c} - \bar{r}') \perp (\bar{c} - \bar{r})$.

P 2-9 Expand $\bar{A} \times (\bar{A} \times \bar{X}) = \bar{A}(\bar{A} \cdot \bar{X}) - \bar{X}(\bar{A} \cdot \bar{A})$

or $\bar{A} \cdot \bar{P} = \phi \bar{A} - A^2 \bar{X}$

$\therefore \bar{X} = \dfrac{1}{A^2}(\phi \bar{A} + \bar{P} \times \bar{A})$.

P 2-10 $\bar{A}_{P_1} = -\bar{a}_y 3 - \bar{a}_z 2$, $\overrightarrow{OP_1} = -\bar{a}_y 2 + \bar{a}_z 3$

$\overrightarrow{OP_2} = \bar{a}_x (r\cos\phi) + \bar{a}_y (r \sin\phi) + \bar{a}_z = \bar{a}_x \frac{\sqrt{3}}{2} - \bar{a}_y \frac{3}{2} + \bar{a}_z$

$\overrightarrow{P_1 P_2} = \overrightarrow{OP_2} - \overrightarrow{OP_1} = \bar{a}_x \frac{\sqrt{3}}{2} + \bar{a}_y \frac{1}{2} - \bar{a}_z 2$, $|\overrightarrow{P_1 P_2}| = \sqrt{5}$

$\bar{A}_{P_1} \cdot \bar{a}_{P_1 P_2} = \bar{A}_{P_1} \cdot \dfrac{\overrightarrow{P_1 P_2}}{|\overrightarrow{P_1 P_2}|} = \dfrac{\sqrt{5}}{2} = 1.12$.

P 2-11 (a) $x = r\cos\phi = 4\cos(2\pi/3) = -2$

$y = r\sin\phi = 4\sin(2\pi/3) = 2\sqrt{3}$

$z = 3$

(b) $R = (r^2 + z^2)^{1/2} = (4^2 + 3^2)^{1/2} = 5$

$\theta = \tan^{-1}(r/z) = \tan^{-1}(4/3) = 53.1°$

$\phi = 2\pi/3 = 120°$.

P 2-12 (a) $\bar{E}_P = \bar{a}_R \dfrac{25}{(-3)^2 + 4^2 + (-5)^2} = \bar{a}_R \dfrac{1}{2}$

$(E_P)_x = \frac{1}{2}\left(-\frac{1}{\sqrt{50}}\right) = 0.212$

(b) $\bar{a}_R = \dfrac{1}{\sqrt{50}}(-\bar{a}_x 3 + \bar{a}_y 4 - \bar{a}_z 5)$, $\bar{a}_A = \dfrac{\bar{B}}{B} = \frac{1}{3}(\bar{a}_x 2 - \bar{a}_y 2 + \bar{a}_z)$

$\theta = \cos^{-1}(\bar{a}_R \cdot \bar{a}_A) = \cos^{-1}\left(-\frac{19}{3\sqrt{50}}\right) = 154°$.

P 2-13 $\bar{a}_A = \bar{a}_x \sin\theta\cos\phi + \bar{a}_y \sin\theta\sin\phi + \bar{a}_z \cos\theta = \dfrac{\bar{a}_x x + \bar{a}_y y + \bar{a}_z z}{\sqrt{x^2 + y^2 + z^2}}$

$\bar{a}_B = \bar{a}_x \cos\theta\cos\phi + \bar{a}_y \cos\theta\sin\phi - \bar{a}_z \sin\theta = \dfrac{\bar{a}_x xz + \bar{a}_y yz - \bar{a}_z(x^2+y^2)}{\sqrt{(x^2+y^2)(x^2+y^2+z^2)}}$

$\bar{a}_\phi = -\bar{a}_x \sin\phi + \bar{a}_y \cos\phi = \dfrac{-\bar{a}_x y + \bar{a}_y x}{\sqrt{x^2 + y^2}}$.

P 2-14 $\int_{P_1}^{P_2} \bar{E} \cdot d\bar{L} = \int_{P_1}^{P_2} (y\,dx + x\,dy)$.

(a) $x = 2y^2$, $dx = 4y\,dy$: $\int_{P_1}^{P_2} \bar{E} \cdot d\bar{L} = \int_1^2 (4y^3 dy + 2y^3 dy) = 14$

(b) $x = 6y - 4$, $dx = 6dy$: $\int_{P_1}^{P_2} \bar{E} \cdot d\bar{L} = \int_1^2 [6y\,dy + (6y - 4)]dy = 14$.

Equal line integrals along two specific paths do not necessarily imply a conservative field. $\bar{E}$ is a conservative field in this case because $\bar{E} = \bar{\nabla}(xy + C)$.

P 2-15 $\begin{bmatrix} E_r \\ E_\phi \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} r\sin\phi \\ r\cos\phi \end{bmatrix}$

$\bar{E} = \bar{a}_r\, r\sin 2\phi + \bar{a}_\phi\, r\cos 2\phi$

$\bar{E} \cdot d\bar{L} = r\sin 2\phi\, dr + r^2 \cos 2\phi\, d\phi$.

$P_3(3, 4, -1) = P_3(5, 53.1°, -1)$; $P_4(4, -3, -1) = P_4(5, -36.9°, -1)$.

There is no change in $r\ (= 5)$ from P_3 to P_4.

$\therefore \int_{P_3}^{P_4} \bar{E} \cdot d\bar{L} = 5^2 \int_{53.1°}^{-36.9°} \cos 2\phi\, d\phi = -24$.

P 2-16 (a) $\bar{\nabla} V = \left[\bar{a}_x \left(\frac{\pi}{3}\cos\frac{\pi}{3}x\right)\left(\sin\frac{\pi}{3}y\right) + \bar{a}_y \left(\sin\frac{\pi}{3}x\right)\left(\frac{\pi}{3}\cos\frac{\pi}{3}y\right) + \bar{a}_z \left(\sin\frac{\pi}{3}x\right)\left(\sin\frac{\pi}{3}y\right)\right] e^{-z}$.

$(\bar{\nabla} V)_P = -\left(\bar{a}_y \frac{\pi}{6} + \bar{a}_z \frac{\sqrt{3}}{2}\right) e^{-1} = -(\bar{a}_y\, 0.026 + \bar{a}_z\, 0.043)$.

(b) $\overrightarrow{PO} = -\bar{a}_x - \bar{a}_y 2 - \bar{a}_z 3$; $\bar{a}_{PO} = -\dfrac{1}{\sqrt{14}}(\bar{a}_x + \bar{a}_y 2 + \bar{a}_z 3)$.

$\therefore (\bar{\nabla} V)_P \cdot \bar{a}_{PO} = \dfrac{1}{\sqrt{14}}\left(\frac{\pi}{3} - \frac{3\pi}{2}\right) e^{-1} = 0.0455$.

P 2-17 $\oint (\bar{a}_R 3\sin\theta) \cdot (\bar{a}_R 5^2 \sin\theta)\, d\theta\, d\phi = \int_0^{2\pi}\int_0^{\pi} 75\sin^3\theta\, d\theta\, d\phi = 75\pi^2$.

P 2-19 $\oint \bar{A} \cdot d\bar{s} = \left(\int_{\text{bottom}} + \int_{\text{top}} + \int_{\text{curved}}\right) \bar{A} \cdot d\bar{s}$.

Top face ($z=4$): $\bar{A} = \bar{a}_r r^2 + \bar{a}_z 8$, $d\bar{s} = \bar{a}_z ds$.

$$\int_{\text{top face}} \bar{A}\cdot d\bar{s} = \int_{\text{top face}} 8\, ds = 8(\pi 5^2) = 200\pi.$$

Bottom face ($z=0$): $\bar{A} = \bar{a}_r r^2$, $d\bar{s} = -\bar{a}_z ds$.

$$\int_{\text{bottom face}} \bar{A}\cdot d\bar{s} = 0.$$

Walls ($r=5$): $\bar{A} = \bar{a}_r 25 + \bar{a}_z 2z$, $ds = \bar{a}_r ds$.

$$\int_{\text{walls}} \bar{A}\cdot d\bar{s} = 25\int_{\text{wall}} ds = 25(2\pi 5\times 4) = 1000\pi.$$

$\therefore \oint \bar{A}\cdot d\bar{s} = 200\pi + 0 + 1000\pi = 1{,}200\pi.$

$\bar{\nabla}\cdot\bar{A} = 3r + 2$, $\int \bar{\nabla}\cdot\bar{A}\, dv = \int_0^4\int_0^{2\pi}\int_0^5 \bar{\nabla}\cdot\bar{A}\, r\, dr\, d\phi\, dz = 1{,}200\pi.$

P.2-20 $\bar{\nabla}\cdot\bar{F} = \frac{1}{r}\frac{\partial}{\partial r}(rF_r) + \frac{\partial}{\partial z}F_z = k_1$, $\int \bar{\nabla}\cdot\bar{F}\, dv = 24\pi k_1$.

Divergence theorem fails because $\bar{F}$ has a singularity at $r=0$.

P.2-21 a) $\oint \bar{D}\cdot d\bar{s} = \oint \left(\frac{\cos^2\phi}{R^2}\right) R^2 \sin\theta\, d\theta\, d\phi$

$$= \int_0^{2\pi}\int_0^{\pi} \left(\frac{1}{2} - 1\right)\cos^2\phi \sin\theta\, d\theta\, d\phi = -\pi.$$

b) $\bar{\nabla}\cdot\bar{D} = -(\cos^2\phi)/R^4$, $\int \bar{\nabla}\cdot\bar{D}\, dv = \int_0^{2\pi}\int_0^{\pi}\int_1^2 (\bar{\nabla}\cdot\bar{D}) R^2 \sin\theta\, dR\, d\theta\, d\phi = -\pi.$

P.2-22 $\bar{\nabla}\cdot\bar{F} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 F_R) = \frac{1}{R^2}\frac{\partial}{\partial R}[R^2 f(R)] = 0.$

$\therefore R^2 f(R) =$ constant, C; i.e., $f(R) = \frac{C}{R^2}$.

P.2-24 a) $\bar{A}\cdot d\bar{L} = 3x^2y^2\, dx - x^3y^3\, dy$

$$\oint \bar{A}\cdot d\bar{L} = 21 + \frac{56}{3} - 7 = \frac{98}{3} = 32\frac{2}{3}.$$

b) $\bar{\nabla}\times\bar{A} = -\bar{a}_z 12x^2y^2$

$$\int (\bar{\nabla}\times\bar{A})\cdot d\bar{s} = \int_1^2\int_y^4 (-\bar{a}_z 12x^2y^2)\cdot(-\bar{a}_z\, dx\, dy) = 32\frac{2}{3}.$$

c) No, because $\bar{\nabla}\times\bar{A} \neq 0$.

P.2-25 $\bar{\nabla}\times\bar{A} = \frac{1}{R\sin\theta}\left(\bar{a}_R \cos\theta \sin\frac{\phi}{2} - \bar{a}_\theta \sin\theta \sin\frac{\phi}{2}\right)$

$$\int_S (\bar{\nabla}\times\bar{A})\cdot d\bar{s} = \int_0^{2\pi}\int_0^{\pi/2} (\bar{\nabla}\times\bar{A})_{R=b}\cdot(\bar{a}_R b^2 \sin\theta\, d\theta\, d\phi) = 4b.$$

$$\oint_C \bar{A}\cdot d\bar{L} = \int_0^{2\pi} (\bar{A})_{\substack{R=b \\ \theta=\pi/2}}\cdot(\bar{a}_\phi\, b\, d\phi) = \int_0^{2\pi} b\sin\frac{\phi}{2}\, d\phi = 4b.$$

Chapter 3

P.3-1 a) $\alpha = \tan^{-1}\left(\frac{L-w}{d_1}\right) = \tan^{-1}\left(\frac{mv_0^2}{eVL_0}\right)$.

b) $d_1 = \frac{d_0}{20}$, $\frac{eE_y}{2m}\frac{w^2}{v_0^2} = \frac{1}{20}\frac{eE_y}{mv_0^2}w\left(L - \frac{w}{2}\right)$, $\frac{L}{w} = 10.5$.

P.3-2 a) Max. voltage V_{max} will make $d_1 = h/2$ at $z = w$.

$$\frac{h}{2} = \frac{e}{2m}\left(\frac{V_{max}}{h}\right)\left(\frac{w}{v_0}\right)^2, \quad \text{or} \quad V_{max} = \frac{m}{e}\left(\frac{v_0 h}{w}\right)^2.$$

b) At the screen, $(d_0)_{max} = D/2$. Hence L must be L_{max}, where

$$L_{max} = \frac{1}{2}\left(w + \frac{mv_0^2 Dh}{eVw}\right).$$

c) Double V_{max} by doubling v_0^2, or doubling the anode accelerating voltage.

P.3-3 $F = \frac{e^2}{4\pi\epsilon_0 R^2} = (9\times10^9)\frac{(1.602\times10^{-19})^2}{(5.28\times10^{-11})^2} = 8.29\times10^{-8}$ (N). Attraction force.

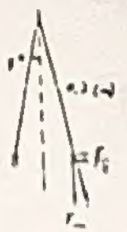
P.3-4 $\overrightarrow{Q_1P} = -\bar{a}_x 2 - \bar{a}_y$; $\overrightarrow{Q_2P} = -\bar{a}_x 3 + \bar{a}_y$.

$$\bar{E}_{P1} = \frac{Q_1}{4\pi\epsilon_0(\sqrt{5})^3}(-\bar{a}_x 2 - \bar{a}_y); \quad \bar{E}_{P2} = \frac{Q_2}{4\pi\epsilon_0(\sqrt{10})^3}(-\bar{a}_x 3 + \bar{a}_y).$$

a) No x-component of $\bar{E}_P$: $-\frac{2Q_1}{(\sqrt{5})^3} - \frac{3Q_2}{(\sqrt{10})^3} = 0$, or $\frac{Q_1}{Q_2} = -\frac{3}{4\sqrt{2}}$.

b) No y-component of $\bar{E}_P$: $-\frac{Q_1}{(\sqrt{5})^3} + \frac{Q_2}{(\sqrt{10})^3} = 0$, or $\frac{Q_1}{Q_2} = \frac{1}{2\sqrt{2}}$.

P.3-5



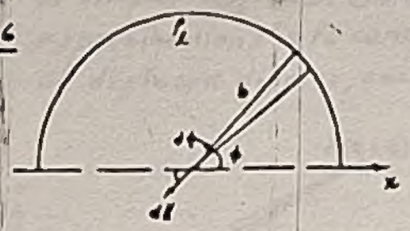
At equilibrium, electric force $\bar{F}_e$ and gravitational force $\bar{F}_m$ must add to give a resultant along the thread.

$$\frac{F_e}{F_m} = \tan 5° = 0.0875.$$

$F_m = mg = 280\times10^{-4}$ (N).

$$F_e = \frac{Q^2}{4\pi\epsilon_0(2\times0.2\sin5°)^2} = 7.41\times10^{-12}Q^2\ (N). \quad Q = 3.40\ (nC).$$

P.3-6

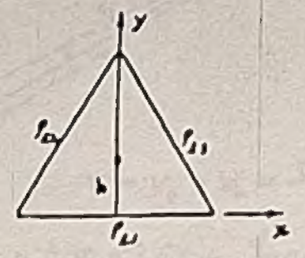


$$dE_y = -\frac{\rho_L (b\,d\phi)}{4\pi\epsilon_0 b^2}\sin\phi.$$

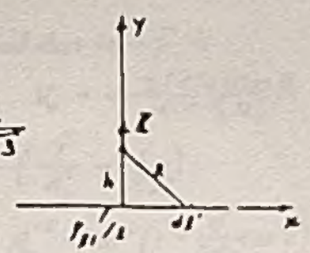
$$\bar{E} = \bar{a}_y E_y = -\bar{a}_y \frac{\rho_L}{4\pi\epsilon_0 b}\int_0^{\pi} \sin\phi\, d\phi$$

$$= -\bar{a}_y \frac{\rho_L}{2\pi\epsilon_0 b}.$$

P.3-7



$h = \frac{L}{2\sqrt{3}}$



$\bar{E}$ at the center of triangle would be zero if all three line charges were of the same charge density. The problem is equivalent to that of a single line charge of density $\rho_{s1}/2$. By symmetry, there will only be a y-component.

$$\bar{E} = \bar{a}_y E_y = \bar{a}_y \int_{-L/2}^{L/2} \frac{(\rho_{s1}/2)\,dL'}{4\pi\epsilon_0 R^2}\left(\frac{h}{R}\right) = \bar{a}_y \int_{-L/2}^{L/2} \frac{\rho_{s1} h\, dL'}{8\pi\epsilon_0 (h^2+L'^2)^{3/2}}$$

$$= \bar{a}_y \frac{3\rho_{s1}}{4\pi\epsilon_0 L} = \bar{a}_y \frac{3\rho_{s1}}{2\pi\epsilon_0 L}.$$

P.3-8 Use Gauss's law: $\oint \bar{E}\cdot d\bar{s} = Q/\epsilon_0$.

a) $\bar{E}$ is normal to the two faces at $x = \pm 0.05$(m), where $E = \pm \bar{a}_x 5$ and $\bar{a}_n = \pm \bar{a}_x$ respectively.

$$Q = 2\epsilon_0 (5 \times 0.1^2) = 0.1\epsilon_0 = 8.84\times10^{-13}\ (C).$$

b) $\bar{E} = \bar{a}_r (100x)\cos\phi - \bar{a}_\phi (100x)\sin\phi = \bar{a}_r (100r\cos^2\phi) - \bar{a}_\phi (100r\sin\phi\cos\phi)$.

$$\oint \bar{E}\cdot\bar{a}_n\, ds = \int_0^{2\pi} (100\times0.05\cos^2\phi)(0.1\times0.05\, d\phi) = 0.025\pi.$$

$$Q = 0.0785\epsilon_0 = 6.94\times10^{-13}\ (C).$$

P.3-9 Spherical symmetry: $\bar{E} = \bar{a}_R E_R$. Apply Gauss's law.

1) $0 \le R \le b$. $4\pi R^2 E_{R1} = \frac{\rho_s}{\epsilon_0}\int_0^R \left(1-\frac{R'^2}{b^2}\right) 4\pi R'^2 dR' = \frac{4\pi\rho_s}{\epsilon_0}\left(\frac{R^3}{3} - \frac{R^5}{5b^2}\right)$.

$$E_{R1} = \frac{\rho_s}{\epsilon_0} R\left(\frac{1}{3} - \frac{R^2}{5b^2}\right).$$

2) $b \le R \le R_1$. $4\pi R^2 \epsilon_0 E_{R2} = \frac{\rho_s}{\epsilon_0}\int_0^b \left(1-\frac{R'^2}{b^2}\right) 4\pi R'^2 dR' = \frac{8\pi\rho_s}{15\epsilon_0} b^3$

$$E_{R2} = \frac{2\rho_s b^3}{15\epsilon_0 R^2}.$$

3) $R_1 \le R \le R_2$. $E_{R3} = 0$.

4) $R > R_2$. $E_{R4} = \frac{2\rho_s b^3}{15\epsilon_0 R^2}$.

P.3-10 Cylindrical symmetry: $\bar{E} = \bar{a}_r E_r$. Apply Gauss's law.

a) $r < a$, $E_r = 0$; $a < r < b$, $E_r = a\rho_{sa}/\epsilon_0 r$;

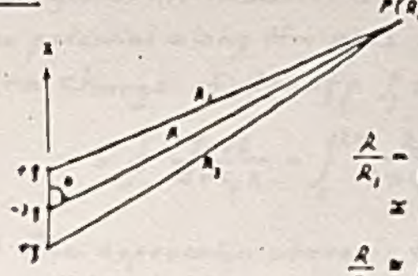
$r > b$: $E_r = (a\rho_{sa} + b\rho_{sb})/\epsilon_0 r$.

b) $b/a = -\rho_{sa}/\rho_{sb}$.

P.3-11 Refer to Eq. (3-49) and Fig. 3-14. $\bar{E}$ will have no z-component if $E_R\cos\theta = E_\theta \sin\theta$, or $2\cos^2\theta = \sin^2\theta$

$\theta = 54.7°$ and $125.3°$.

P.3-12



$$V = \frac{q}{4\pi\epsilon_0 R}\left(\frac{R}{R_1} + \frac{R}{R_2} - 2\right).$$

$$R_1^2 = R^2 + \left(\frac{d}{2}\right)^2 - 2Rd\cos\theta.$$

$$\frac{R}{R_1} = \left[1 + \left(\frac{d}{2R}\right)^2 - \frac{d}{R}\cos\theta\right]^{-1/2}$$

$$\approx 1 + \frac{d}{2R}\cos\theta + \frac{d^2}{4R^2}\frac{3\cos^2\theta-1}{2}.$$

$$\frac{R}{R_2} \approx 1 - \frac{d}{2R}\cos\theta + \frac{d^2}{4R^2}\frac{3\cos^2\theta-1}{2}.$$

a) $\therefore V = \frac{q(d/2)^2}{4\pi\epsilon_0 R}(3\cos^2\theta - 1)$, $R^2 \gg d^2$.

$$\bar{E} = -\bar{\nabla}V = -\bar{a}_R\frac{\partial V}{\partial R} - \bar{a}_\theta\frac{\partial V}{R\partial\theta} = \frac{q(d/2)^2}{4\pi\epsilon_0 R^4}\left[\bar{a}_R\, 2(3\cos^2\theta-1) + \bar{a}_\theta\, 3\sin 2\theta\right].$$

b) Equation for equipotential surfaces:

$$R = C_1(3\cos^2\theta - 1)^{1/3}.$$

Equation for streamlines:

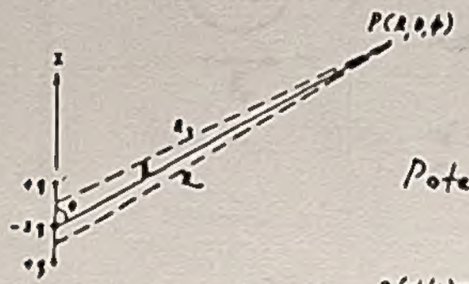
$$\frac{dR}{E_R} = \frac{R\,d\theta}{E_\theta}, \text{ or } \frac{dR}{3\cos^2\theta - 1} = \frac{R\,d\theta}{3\sin 2\theta}$$

$$\frac{dR}{R} = \frac{3\,d(\sin\theta)}{3\sin\theta} - \frac{d\theta}{\sin 2\theta}$$

$$R = C_2\left(\sin^{1/3}\theta/\sqrt{|\cos\theta|}\right).$$

(Also see next page)

P. 3-12 A simpler approach (not to the same degree of approximation) is to consider the problem as a pair of displaced dipoles, each with a moment $\bar{p} = q\bar{d}/2$.



From Eq. (3-48), potential due to $\bar{p}_+$ is

$$V_+ = \frac{q(d/2)\cos\theta}{4\pi\epsilon_0 R_+^2}.$$

Potential due to $-\bar{p}_-$ is

$$V_- = -\frac{q(d/2)\cos\theta}{4\pi\epsilon_0 R_-^2}.$$

$$V = V_+ + V_- = \frac{q(d/2)\cos\theta}{4\pi\epsilon_0}\left(\frac{1}{R_+^2} - \frac{1}{R_-^2}\right).$$

$$R_+^{-2} \cong R^{-2}\left(1 + \frac{d}{2R}\cos\theta\right), \quad R_-^{-2} \cong R^{-2}\left(1 - \frac{d}{2R}\cos\theta\right).$$

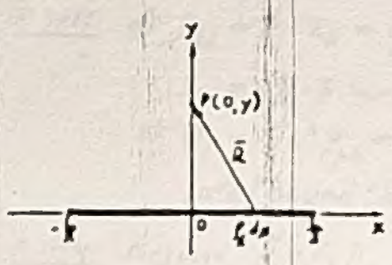
a) $$\therefore V = \frac{q(d\cos\theta)^2/2}{4\pi\epsilon_0 R^3}.$$

$$\bar{E} = -\nabla V = \frac{qd^3/2}{4\pi\epsilon_0 R^4}\left(\bar{a}_R 3\cos^2\theta + \bar{a}_\theta \sin 2\theta\right).$$

b) Equation for equipotential surfaces: $R = C_v'(\cos\theta)^{2/3}$

Equation for streamlines: $R = C_s'(\sin\theta)^{1/2}$

P. 3-13



a) $$V = 2\int_0^{L/2} \frac{\rho_L\, dx}{4\pi\epsilon_0 R}$$

$$= \frac{\rho_L}{2\pi\epsilon_0}\int_0^{L/2} \frac{dx}{\sqrt{x^2+y^2}}$$

$$= \frac{\rho_L}{2\pi\epsilon_0}\left\{\ln\left[\sqrt{\left(\frac{L}{2}\right)^2 + y^2} + \frac{L}{2}\right] - \ln y\right\}.$$

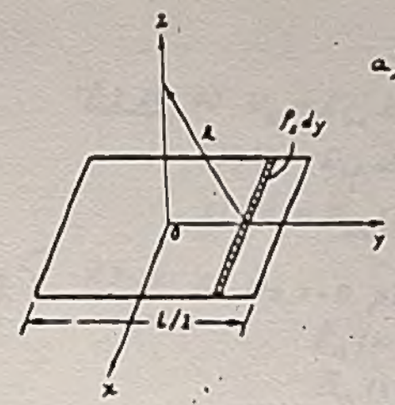
b) From Coulomb's law:

$$\bar{E} = \bar{a}_y E_y = 2\int_0^{L/2} \bar{a}_y \frac{\rho_L\, y\, dx}{4\pi\epsilon_0 R^3} = \bar{a}_y \frac{\rho_L}{2\pi\epsilon_0 y} \frac{L/2}{\sqrt{(L/2)^2 + y^2}}.$$

c) $E = -\nabla V$ gives the same answer.

P. 3-14 Surface charge density $\rho_s = \frac{Q}{L^2}$.

Use the results of problem P. 3-13 for the coordinate system chosen in the figure on the next page. Replace ρ_L by $\rho_s dy$ and y by $\sqrt{y^2+x^2}$.



a) $$V = 2\cdot\frac{\rho_s}{2\pi\epsilon_0}\int_0^{L/2}\left\{\ln\left[\sqrt{\left(\frac{L}{2}\right)^2 + y^2 + x^2} + \left(\frac{L}{2}\right)\right] - \ln\sqrt{y^2+x^2}\right\} dy$$

$$= \frac{Q}{\pi\epsilon_0 L^2}\left\{\frac{L}{2}\ln\left[\frac{\sqrt{2\left(\frac{L}{2}\right)^2 + x^2} + \frac{L}{2}}{\sqrt{2\left(\frac{L}{2}\right)^2 + x^2} - \frac{L}{2}}\right] - 2x\tan^{-1}\left[\frac{\left(\frac{L}{2}\right)^2}{x\sqrt{2\left(\frac{L}{2}\right)^2 + x^2}}\right]\right\}.$$

b) $$\bar{E} = -\bar{\nabla}V = \frac{2Q}{\pi\epsilon_0 L^2}\tan^{-1}\left[\frac{\left(\frac{L}{2}\right)^2}{x\sqrt{2\left(\frac{L}{2}\right)^2 + x^2}}\right].$$

P. 3-15 Assume the circular tube sits on the xy-plane with its axis coinciding with the x-axis. The surface charge on the tube wall is $\rho_s = Q/2\pi bh$. First find the potential along the axis at x due to a circular line charge of density ρ_L situated at x':

$$V = \oint \frac{\rho_L\, dl}{4\pi\epsilon_0 R} = \int_0^{2\pi} \frac{\rho_L\, b\, d\phi}{4\pi\epsilon_0\sqrt{b^2 + (x-x')^2}} = \frac{\rho_L\, b}{2\epsilon_0\sqrt{b^2 + (x-x')^2}}.$$

a) The expression above is the contribution dV due to a circular line charge of density $\rho_L = \rho_s\, dx'$.

$$dV = \frac{\rho_s\, b\, dx'}{2\epsilon_0\sqrt{b^2 + (x-x')^2}}.$$

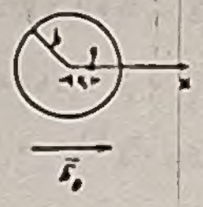
At a point outside the tube:

$$V = \int_{x'=0}^{x'=h} dV = \frac{b\rho_s}{2\epsilon_0}\ln\frac{x + \sqrt{b^2+x^2}}{(x-h) + \sqrt{b^2 + (x-h)^2}}$$

$$\bar{E} = -\bar{a}_x\frac{dV}{dx} = \bar{a}_x\frac{b\rho_s}{2\epsilon_0}\left[\frac{1}{\sqrt{b^2 + (x-h)^2}} - \frac{1}{\sqrt{b^2+x^2}}\right].$$

b) Same expressions are obtained for V and $\bar{E}$ at a point inside the tube.

P.3-16



Applied $\bar{E}_0$ causes a displacement r_0.

Force of separation: qE_0;

Restoring force (attraction): qE_n.

E_n at q due to spherical volume of electrons of radius r_0 is (by Gauss's law) $E_n = \frac{\rho' r}{3\epsilon_0} = -\frac{\rho}{3\epsilon_0}|r|$

$$|r| = \frac{3}{\frac{4}{3}\pi b^3} = \frac{3N|e|}{4\pi b^3}.$$

At equilibrium: $E_0 = |E_n| = \frac{3N|e|}{4\pi\epsilon_0 b^3}$, or $r_0 = \frac{4\pi\epsilon_0 b^3}{N|e|}E_0$.

P.3-17 $W = -q\int \bar{E}\cdot d\bar{\ell} = -q\int (y\,dx + x\,dy)$.

a) Along the parabola $x = 2y^2$; $dx = 4y\,dy$

$$W = -q\int_1^2 6y^2\,dy = -14q = 28\ (\mu J).$$

b) Along the straight line $x = 6y - 4$; $dx = 6\,dy$

$$W = -q\int_1^2 (12y-4)\,dy = -14q = 28\ (\mu J).$$

P.3-18 a) $\rho_{ps} = \bar{P}\cdot\bar{a}_n = P_0\frac{L}{2}$ on all six faces of the cube.

$\rho_p = -\nabla\cdot\bar{P} = -3P_0$.

b) $Q_s = 6L^2\rho_{ps} = 3P_0L^3$, $\quad Q_v = L^3\rho_p = -3P_0L^3$.

Total bound charge $= Q_s + Q_v = 0$.

P.3-19 Assume $\bar{P} = \bar{a}_z P$. Surface charge density $\rho_{ps} = \bar{P}\cdot\bar{a}_n = (\bar{a}_z P)\cdot(\bar{a}_n) = P\cos\theta$.

The z-component of the electric field intensity due to a ring of ρ_{ps} contained in width $R\,d\theta$ at θ is

$$dE_z = \frac{P\cos\theta}{4\pi\epsilon_0 R^2}(2\pi R\sin\theta)(R\,d\theta)\cos\theta = \frac{P}{2\epsilon_0}\cos^2\theta\sin\theta\,d\theta.$$

At the center of the cavity: $\bar{E} = \bar{a}_z E_z = \bar{a}_z\frac{P}{2\epsilon_0}\int_0^\pi \cos^2\theta\sin\theta\,d\theta = \frac{\bar{P}}{3\epsilon_0}$.

P.3-20 a) $V_b = E_{ba}\,d = 3\times 50 = 150\ (kV)$

b) $V_b = E_{bp}\,d = 20\times 50 = 1{,}000\ (kV)$

c) $V_b = E_{ba}(d-d_p) + \frac{1}{3}E_{ba}d_p = 3(40 + \frac{1}{3}\times 10) = 130\ (kV)$.

25

P.3-21 At the $z=0$ plane: $\bar{E}_1 = \bar{a}_x 2y - \bar{a}_y 3x + \bar{a}_z 5$.

$$\bar{E}_{1t}(z=0) = \bar{E}_{2t}(z=0) = \bar{a}_x 2y - \bar{a}_y 3x.$$

$$\bar{D}_{1n}(z=0) = \bar{D}_{2n}(z=0) \longrightarrow 2\bar{E}_{1n}(z=0) = 3\bar{E}_{2n}(z=0)$$

$$\longrightarrow \bar{E}_{2n}(z=0) = \tfrac{2}{3}(5) = \bar{a}_z\tfrac{10}{3}.$$

$$\therefore \bar{E}_2(z=0) = \bar{a}_x 2y - \bar{a}_y 3x + \bar{a}_z\tfrac{10}{3}.$$

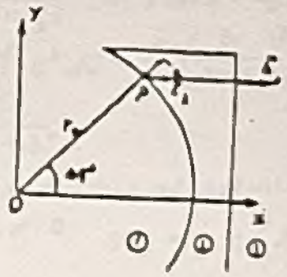
$$\bar{D}_2(z=0) = (\bar{a}_x 6y - \bar{a}_y 9x + \bar{a}_z 10)\epsilon_0.$$

P.3-22 $\bar{P}_1 = \epsilon_0(\epsilon_r - 1)\bar{E}_1$: $\bar{E}_{t1} = \bar{E}_{t2} \longrightarrow \frac{1}{\epsilon_{r1}-1}P_{t1} = \frac{1}{\epsilon_{r2}-1}P_{t2}$.

$$\epsilon_{n1}\bar{E}_{n1} = \epsilon_{n2}\bar{E}_{n2} \longrightarrow \frac{\epsilon_{n1}}{\epsilon_{r1}-1}P_{n1} = \frac{\epsilon_{n2}}{\epsilon_{r2}-1}P_{n2}.$$

P.3-23 $\epsilon_1\frac{\partial V_1}{\partial n} = \epsilon_2\frac{\partial V_2}{\partial n}$ and $V_1 = V_2$.

P.3-24



Assume $\bar{E}_2 = \bar{a}_r E_{2r} + \bar{a}_\phi E_{2\phi}$

B.C.: $\bar{a}_n\times\bar{E}_1 = \bar{a}_n\times\bar{E}_2 \longrightarrow E_{2\phi} = -3$.

For $\bar{E}_1$, and hence $\bar{E}_2$, to be parallel to the x-axis,

$E_{2\phi} = -E_{2r} \longrightarrow E_{2r} = 3$.

B.C.: $\bar{a}_n\cdot\bar{D}_1 = \bar{a}_n\cdot\bar{D}_2 \longrightarrow 5 = 3\epsilon_{r2}$

$\therefore \epsilon_{r2} = 5/3$.

P.3-25 $\epsilon = \frac{\epsilon_2-\epsilon_1}{d}y + \epsilon_1$.

Assume Q on plate at $y=d$. $\bar{E} = -\bar{a}_y\frac{\rho_s}{\epsilon} = -\bar{a}_y\frac{Q}{S(\frac{\epsilon_2-\epsilon_1}{d}y+\epsilon_1)}$.

$$V = -\int_{y\,0}^{y\,d}\bar{E}\cdot d\bar{\ell} = \frac{Qd\ln(\epsilon_2/\epsilon_1)}{S(\epsilon_2-\epsilon_1)},$$

$$C = \frac{Q}{V} = \frac{S(\epsilon_2-\epsilon_1)}{d\ln(\epsilon_2/\epsilon_1)}.$$

P.3-26 a) $C = 4\pi\epsilon_0 R = \frac{1}{9}\times 10^{-9}\times(6.37\times 10^6) = 7.08\times 10^{-4}\ (F)$.

b) $E_b = 3\times 10^6 = \frac{Q_{max}}{4\pi\epsilon_0 R^2}$. $Q_{max} = 1.35\times 10^{10}\ (C)$.

P 3-27 Assume charge Q on conducting sphere.

$b<R<b+d$, $\bar{E}_1 = \bar{a}_R \frac{Q}{4\pi\epsilon_0(1+X_e)R^2}$.

$R>b+d$, $\bar{E}_2 = \bar{a}_R \frac{Q}{4\pi\epsilon_0 R^2}$.

$V = -\int_\infty^b \bar{E}\cdot d\bar{L} = -\int_\infty^{b+d} E_2\,dR - \int_{b+d}^b E_1\,dR = \frac{Q}{4\pi\epsilon_0(1+X_e)}\left(\frac{X_e}{b+d}+\frac{1}{b}\right)$.

$C = \frac{Q}{V} = \frac{4\pi\epsilon_0(1+X_e)}{\frac{X_e}{b+d}+\frac{1}{b}}$.

P 3-28 Assume charge Q on inner shell, -Q on outer shell.

$R_i<R<R_o$, $\bar{D} = \bar{a}_R \frac{Q}{4\pi R^2}$.

$R_i<R<b$, $\bar{E}_1 = \frac{\bar{D}}{\epsilon_0\epsilon_r}$; $b<R<R_o$, $\bar{E}_2 = \frac{\bar{D}}{2\epsilon_0\epsilon_r}$.

$V = -\int_{R_o}^{R_i} \bar{E}\cdot d\bar{R} = -\int_b^{R_i} E_1\,dR - \int_{R_o}^b E_2\,dR = \frac{Q}{4\pi\epsilon_0\epsilon_r}\left(\frac{1}{R_i}-\frac{1}{2b}-\frac{1}{2R_o}\right)$.

a) $\bar{D} = \bar{a}_R \frac{\epsilon_0\epsilon_r V}{R^2\left(\frac{1}{R_i}-\frac{1}{2b}-\frac{1}{2R_o}\right)}$, $R_i<R<R_o$. $\bar{D}=0$, $\bar{E}=0$ for $R<R_i$ and $R>R_o$.

$\bar{E}_1 = \bar{a}_R \frac{V}{R^2\left(\frac{1}{R_i}-\frac{1}{2b}-\frac{1}{2R_o}\right)}$; $\bar{E}_2 = \bar{a}_R \frac{V}{R^2\left(\frac{1}{R_i}-\frac{1}{b}-\frac{1}{R_o}\right)}$.

b) $C = \frac{Q}{V} = \frac{4\pi\epsilon_0\epsilon_r}{\frac{1}{R_i}-\frac{1}{2b}-\frac{1}{2R_o}}$.

P 3-29 Let ρ_L be the linear charge density on the inner conductor.

$\bar{E} = \bar{a}_r \frac{\rho_L}{2\pi\epsilon r}$, $V_0 = -\int_b^a \bar{E}\cdot d\bar{r} = \frac{\rho_L}{2\pi\epsilon}\ln\left(\frac{b}{a}\right)$.

$\rho_L = \frac{2\pi\epsilon V_0}{\ln(b/a)}$.

a) $\bar{E}(a) = \bar{a}_r \frac{V_0}{a\ln(b/a)}$.

b) Let $x = b/a$, $f(x) = \frac{\ln x}{x}$, $\frac{df(x)}{dx} = 0 \longrightarrow \ln x = 1$, $x = \frac{b}{a} = 2.718$.

c) $C = \frac{\rho_L}{V_0} = \frac{2\pi\epsilon}{\ln(b/a)} = 2\pi\epsilon$ (F/m).

P 3-30 $\bar{D} = \bar{a}_r \frac{\rho_L}{2\pi r}$. $\epsilon_1 = \bar{a}_r \frac{\rho_L}{2\pi\epsilon_0\epsilon_{r1}r}$, $r_1<r<b$; $\bar{E}_2 = \bar{a}_r \frac{\rho_L}{2\pi\epsilon_0\epsilon_{r2}r}$, $b<r<r_0$.

$V = -\int_{r_0}^{r_1} \bar{E}\cdot d\bar{r} = \frac{\rho_L}{2\pi\epsilon_0}\left[\frac{1}{\epsilon_{r1}}\ln\left(\frac{b}{r_1}\right)+\frac{1}{\epsilon_{r2}}\ln\left(\frac{r_0}{b}\right)\right]$.

$C = \frac{\rho_L}{V} = \frac{2\pi\epsilon_0}{\frac{1}{\epsilon_{r1}}\ln\left(\frac{b}{r_1}\right)+\frac{1}{\epsilon_{r2}}\ln\left(\frac{r_0}{b}\right)}$ (F/m).

P 3-31 From Gauss's law, $\oint \bar{D}\cdot d\bar{s} = \rho_L L$.

$\bar{E}_1 = \bar{E}_2 = \bar{a}_r E_r$. $\pi r L(\epsilon_0\epsilon_{r1}+\epsilon_0\epsilon_{r2})E_r = \rho_L L$.

$E_r = \frac{\rho_L}{\pi r\epsilon_0(\epsilon_{r1}+\epsilon_{r2})}$; $V = -\int_{r_0}^{r_1} E_r\,dr = \frac{\rho_L}{\pi\epsilon_0(\epsilon_{r1}+\epsilon_{r2})}\ln\left(\frac{r_0}{r_1}\right)$.

$\therefore C = \frac{\rho_L L}{V} = \frac{\pi\epsilon_0(\epsilon_{r1}+\epsilon_{r2})L}{\ln(r_0/r_1)}$.

P 3-32 $\bar{E} = \bar{a}_r \frac{\rho_L}{2\pi\epsilon r} = \bar{a}_r \frac{\rho_L}{2\pi\epsilon_0(2+\frac{1}{r})r} = \bar{a}_r \frac{\rho_L}{4\pi\epsilon_0(r+2)}$.

$V = -\int_{r_0}^{r_1} \bar{E}\cdot d\bar{r} = \frac{\rho_L}{4\pi\epsilon_0}\ln(r+2)\Big|_{r_0}^{r_1} = \frac{\rho_L}{4\pi\epsilon_0}\ln\left(\frac{9}{7}\right)$.

$C = \frac{\rho_L L}{V} = \frac{4\pi L\epsilon_0}{\ln(9/7)} = 1500\epsilon_0 = 13.26$ (μF).

P 3-33 $\bar{D} = \bar{a}_R \frac{R}{3}\rho$, $R<b$; $\bar{D} = \bar{a}_R \frac{b^3\rho}{3R^2}$, $R>b$; $\bar{E} = \frac{1}{\epsilon_0}\bar{D}$.

a) $W_i = \frac{1}{2}\int_v \bar{D}\cdot\bar{E}\,dv = \frac{1}{2}\int_0^b \frac{1}{\epsilon_0}\left(\frac{R}{3}\rho\right)^2 4\pi R^2 dR = \frac{2\pi b^5\rho^2}{45\epsilon_0}$.

b) $W_o = \frac{1}{2}\int_b^\infty \frac{1}{\epsilon_0}\left(\frac{b^3\rho}{3R^2}\right)^2 4\pi R^2 dR = \frac{2\pi b^5\rho^2}{9\epsilon_0}$.

Total $W = W_i + W_o = \frac{4\pi b^5\rho^2}{15\epsilon_0}$.

P 3-34 $\bar{E} = \frac{p}{4\pi\epsilon_0 R^3}(\bar{a}_R 2\cos\theta + \bar{a}_\theta \sin\theta)$.

$W = \frac{1}{2}\epsilon_0\int_v E^2\,dv = \frac{\epsilon_0}{2}\left(\frac{p}{4\pi\epsilon_0}\right)^2\int_0^{2\pi}\int_0^\pi d\phi\,d\theta\int_b^\infty \frac{1}{R^4}(4\cos^2\theta+\sin^2\theta)R^2\sin\theta\,dR$

$= \frac{p^2}{12\pi\epsilon_0 b^3}$.

P 3-35 Two conductors at potentials V_1 and V_2 carrying charges $+Q$ and $-Q$: $W_e = \frac{1}{2}V_1\int_{S_1}\rho_s\,ds + \frac{1}{2}V_2\int_{S_2}\rho_s\,ds = \frac{1}{2}Q(V_1-V_2)$

$= \frac{1}{2}CV^2$, $V = V_1 - V_2$.

P 3-36 a) Region 1 — dielectric; region 2 — air.

$\bar{E}_1 = -\bar{a}_y\frac{V_0}{d}$, $\bar{D}_1 = -\bar{a}_y\epsilon_0\epsilon_r\frac{V_0}{d}$, $\rho_{s1} = \epsilon_0\epsilon_r\frac{V_0}{d}$ (top plate)

$\bar{E}_2 = -\bar{a}_y\frac{V_0}{d}$, $\bar{D}_2 = -\bar{a}_y\epsilon_0\frac{V_0}{d}$, $\rho_{s2} = \epsilon_0\frac{V_0}{d}$ (top plate).

$\frac{W_{e1}}{W_{e2}} = \frac{\epsilon_r x}{L-x} = 1 \longrightarrow x = \frac{L}{\epsilon_r+1}$.

P 3-37 $\overset{+Q}{\bullet}\quad\overset{-Q}{\bullet}\longrightarrow x$, a. From Eqs. (3-136) and (3-137), $W_e = -\frac{Q^2}{4\pi\epsilon_0 x}$.

$x = x_+ - x_-$. Force on $+Q$, $\bar{F}_{e+} = -\bar{a}_x\frac{\partial W_e}{\partial x_+} = -\bar{a}_x\frac{Q^2}{4\pi\epsilon_0 x^2} = -\bar{F}_{e-} = -(-\bar{a}_x\frac{Q^2}{4\pi\epsilon_0 x^2})$.

P 3-18 a) $V = V_0 = \text{constant}$. $C = \frac{\epsilon x + \epsilon_1(l-x)}{d} w$.

$W_e = \frac{1}{2} C V_0^2$, $\bar{F}_v = \bar{\nabla} W_e = \bar{a}_x \frac{V_0^2 w}{2d} (\epsilon - \epsilon_1)$.

b) $Q = \text{constant} = C V_0$.

$W_e = \frac{Q^2}{2C}$, $\bar{F}_e = -\bar{\nabla} W_e = \frac{Q^2 d}{2} \frac{\bar{a}_x (\epsilon - \epsilon_1) w}{[\epsilon x + \epsilon_1 (l-x)]^2}$

$= \bar{a}_x \frac{V_0^2 w}{2d} (\epsilon - \epsilon_1)$.

Chapter 4

P 4-1 $\bar{\nabla}^2 V = 0 \longrightarrow V_d = c_1 y + c_2$, $\bar{E}_d = -\bar{a}_y c_1$, $\bar{D}_d = -\bar{a}_y \epsilon_0 \epsilon_1 c_1$.

$V_a = c_3 y + c_4$, $\bar{E}_a = -\bar{a}_y c_3$, $\bar{D}_a = -\bar{a}_y \epsilon_0 c_3$.

B.C.: $V_d = 0$ at $y = 0$; $V_a = V_0$ at $y = d$;

$V_d = V_a$ at $y = 0.8d$; $\bar{D}_d = \bar{D}_a$ at $y = 0.8d$.

Solving: $c_1 = \frac{V_0}{(0.8 + 0.2\epsilon_1)d}$, $c_2 = 0$, $c_3 = \frac{\epsilon_1 V_0}{(0.8 + 0.2\epsilon_1)d}$, $c_4 = \frac{(1-\epsilon_1)V_0}{1 + 0.8\epsilon_1}$.

a) $V_d = \frac{5 y V_0}{(4 + \epsilon_1)d}$, $\bar{E}_d = -\bar{a}_y \frac{5 V_0}{(4 + \epsilon_1)d}$.

b) $V_a = \frac{5\epsilon_1 y - 4(\epsilon_1 - 1)d}{(4 + \epsilon_1)d} V_0$, $\bar{E}_a = -\bar{a}_y \frac{5\epsilon_1 V_0}{(4 + \epsilon_1)d}$.

c) $(\rho_s)_{y=d} = -(D_a)_{y=d} = -\frac{5\epsilon_0 \epsilon_1 V_0}{(4 + \epsilon_1)d}$.

$(\rho_s)_{y=0} = (D_d)_{y=0} = -\frac{5\epsilon_1 \epsilon_1 V_0}{(4 + \epsilon_1)d}$.

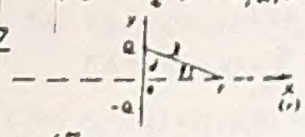
P 4-3 At a point where V is a maximum (minimum) the second partial derivatives of V with respect to x, y and z would all be negative (positive); their sum could not vanish, as required by Laplace's equation.

P 4-6 $\bar{\nabla}^2 V = -\frac{A}{\epsilon r} \longrightarrow \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial V}{\partial r}\right) = -\frac{A}{\epsilon r}$.

$V = -\frac{A}{\epsilon} r + c_1 \ln r + c_2$ $\quad c_1 = \frac{\frac{A}{\epsilon}(b-a) - V_0}{\ln(b/a)}$.

$\begin{cases} V_0 = -\frac{A}{\epsilon} a + c_1 \ln a + c_2 \\ 0 = -\frac{A}{\epsilon} b + c_1 \ln b + c_2 \end{cases}$ $\quad c_2 = \frac{V_0 \ln b + \frac{A}{\epsilon}(a \ln b - b \ln a)}{\ln(b/a)}$.

P 4-7



$\bar{E}\big|_{y=0} = -\bar{a}_y \frac{Q}{4\pi\epsilon R^2} 2\sin\theta = -\bar{a}_y \frac{ad}{2\pi\epsilon(d^2+r^2)^{3/2}}$.

a) $\rho_s = \bar{a}_y \cdot \epsilon \bar{E}\big|_{y=0} = -\frac{Qd}{2\pi(d^2+r^2)^{3/2}}$.

b) $\int_0^\infty \rho_s 2\pi r dr = -Q$.

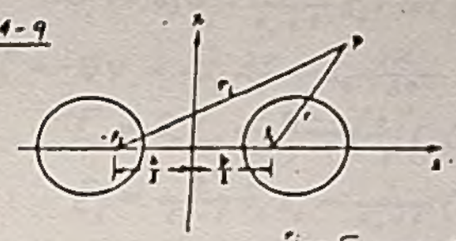
P 4-8 a) Original point charge Q at $y = d/3$ and images

Q at $y = (1+6n)d/3$, $n = \pm 1, \pm 2, \cdots$

$-Q$ at $y = (5+6n)d/3$, $n = 0, \pm 1, \pm 2, \cdots$

b) Original line charge ρ_L at $\theta = 30°$ and image line charges ρ_L at $\theta = 150°$ and $-90°$; $-\rho_L$ at $\theta = 90°, -30°$ and $+150°$.

P 4-9



From Eq. (4-40), $V = \frac{\rho_L}{2\pi\epsilon_0} \ln \frac{r_1}{r}$.

$r = [x^2 + (z - \frac{1}{3})^2]^{1/2}$.

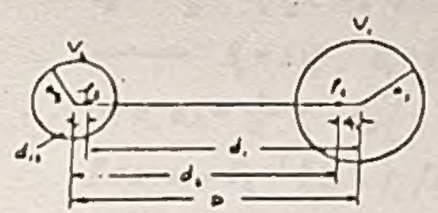
$r_1 = [x^2 + (z + \frac{1}{3})^2]^{1/2}$.

For equipotential surfaces,

$\frac{r_1}{r} = \sqrt{K} \longrightarrow x^2 + [z - (\frac{K+1}{K-1})\frac{1}{3}]^2 = \frac{8^2 K}{(K-1)^2}$.

P 4-10

$V_1 = -\frac{\rho_L}{2\pi\epsilon_0} \ln \frac{a_1}{d_1}$, $V_2 = +\frac{\rho_L}{2\pi\epsilon_0} \ln \frac{a_2}{d_2}$.



Capacitance per unit length

$C = \frac{\rho_L}{V_1 - V_2} = \frac{2\pi\epsilon_0}{\ln \frac{d_1 d_2}{a_1 a_2}}$.

Four equations:

$a_1^2 = d_{11} d_1$, $a_2^2 = d_{11} d_2$

$d_1 + d_{11} = D$, $d_2 + d_{11} = D$.

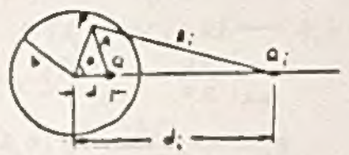
We obtain

$\frac{d_1 d_2}{a_1 a_2} = \frac{a_1 a_2}{d_{11} d_{11}}$ and $a_1^2 + a_2^2 + d_1 d_2 + d_{11} d_{11} = D^2$.

$\frac{d_1 d_2}{a_1 a_2} = \frac{D^2}{2 a_1 a_2} - \frac{a_1}{2 a_2} - \frac{a_2}{2 a_1} + \sqrt{\left(\frac{D^2}{2 a_1 a_2} - \frac{a_1}{2 a_2} - \frac{a_2}{2 a_1}\right)^2 - 1}$.

$\therefore C = \frac{2\pi\epsilon_0}{\ln\left|\frac{1}{2}\left(\frac{D^2}{a_1 a_2} - \frac{a_1}{a_2} - \frac{a_2}{a_1}\right) - \sqrt{\frac{1}{4}\left(\frac{D^2}{a_1 a_2} - \frac{a_1}{a_2} - \frac{a_2}{a_1}\right)^2 - 1}\right|} = \frac{2\pi\epsilon_0}{\cosh^{-1}\left|\frac{1}{2}\left(\frac{D^2}{a_1 a_2} - \frac{a_1}{a_2} - \frac{a_2}{a_1}\right)\right|}$.

4-12



$Q_i = -\frac{b}{d} Q$, $d_i = \frac{b^2}{d}$.

a) $V = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{R} - \frac{b}{d R_i}\right)$.

b) $\rho_s = -\epsilon_0 \frac{\partial V}{\partial \lambda}\Big|_{r=a}$

$= -\frac{Q(b^2 - d^2)}{4\pi a (b^2 + d^2 - 2bd\cos\theta)^{3/2}}$.

P.4-13 At the plane boundary $x=0$: $V_1 = V_2$ and $\epsilon_1 \frac{\partial V_1}{\partial x} = \epsilon_2 \frac{\partial V_2}{\partial x}$.

Two equations: $\frac{Q-Q_1}{\epsilon_1} = \frac{Q_1+Q}{\epsilon_2}$ and $Q+Q_1 = Q_2+Q$.

$\longrightarrow$ $Q_1 = Q_2 = \frac{\epsilon_2-\epsilon_1}{\epsilon_2+\epsilon_1} Q$.

P.4-14 The solution $V(x,y) = V_0$ satisfies $\nabla^2 V = 0$ and all boundary conditions. $\longrightarrow$ Unique solution.

P.4-16 $V(x,y) = \sum_n \sin\frac{n\pi}{a}x \left[A_n \sinh\frac{n\pi}{a}y + B_n \cosh\frac{n\pi}{a}y\right]$.

At $y=0$, $V(x,0) = V_1 = \sum_n B_n \sin\frac{n\pi}{a}x \longrightarrow B_n = \begin{cases} \frac{4V_1}{n\pi}, & n \text{ odd} \\ 0, & n \text{ even.} \end{cases}$

At $y=b$, $V(x,b) = V_1 = \sum_n \sin\frac{n\pi}{a}x \left[A_n \sinh\frac{n\pi b}{a} + B_n \cosh\frac{n\pi b}{a}\right]$

$\longrightarrow A_n \sinh\frac{n\pi b}{a} + B_n \cosh\frac{n\pi b}{a} = \begin{cases} \frac{4V_1}{n\pi}, & n \text{ odd} \\ 0, & n \text{ even.} \end{cases}$

$\therefore A_n = \begin{cases} \frac{4}{n\pi \sinh\frac{n\pi b}{a}} \left(V_1 - V_2 \cosh\frac{n\pi b}{a}\right), & n \text{ odd} \\ 0, & n \text{ even.} \end{cases}$

P.4-18 The solution is the superposition of that for Example 4-9 and that for Fig. 4-12 rotated 90° in the clockwise direction. (In both cases V_0 should be replaced by $V_0/2$.)

Inside: $V(r,\phi) = \frac{2V_0}{\pi} \sum_{n=odd} \frac{1}{n} \left(\frac{r}{b}\right)^n \left[\sin n\phi + \sin n(\phi+\frac{\pi}{2})\right]$, $r<b$

Outside: $V(r,\phi) = -\frac{2V_0}{\pi} \sum_{n=odd} \frac{1}{n} \left(\frac{b}{r}\right)^n \left[\sin n\phi + \sin n(\phi+\frac{\pi}{2})\right]$, $r>b$.

P.4-19 $V(r,\phi) = -E_0 r\cos\phi + \sum_{n=1}^{\infty} B_n r^{-n} \cos n\phi$. $\left(\text{At } r>>b,\ \bar{E} = \bar{a}_r E_0,\ V = -E_0 r\cos\phi\right)$

At $r=b$, $V(b,\phi) = -E_0 b\cos\phi + \sum_{n=1}^{\infty} B_n b^{-n} \cos n\phi$

$\longrightarrow B_1 = E_0 b^2$; $B_n = 0$ for $n \neq 1$.

Outside the cylinder: $V(r,\phi) = -E_0 r\left(1 - \frac{b^2}{r^2}\right)\cos\phi$.
$r>b$

$\bar{E}(r,\phi) = -\bar{\nabla}V = \bar{a}_r E_0 \left(\frac{b^2}{r^2}+1\right)\cos\phi + \bar{a}_\phi E_0 \left(\frac{b^2}{r^2}-1\right)\sin\phi$.

P.4-10 For $r \geq b$, $V_2(r,\phi) = -E_0 r\cos\phi + \sum_{n=1}^{\infty} B_n r^{-n} \cos n\phi$,

$r \leq b$, $V_1(r,\phi) = \sum_{n=1}^{\infty} A_n r^n \cos n\phi$.

At $r=b$: $V_2(b,\phi) = V_1(b,\phi) \longrightarrow -E_0 b + B_1 b^{-1} = A_1 b$; $B_n b^{-n} = A_n b^n$, $n \neq 1$.

$-\frac{\partial V_2}{\partial r}\Big|_{r=b} = -\epsilon_r \frac{\partial V_1}{\partial r}\Big|_{r=b} \longrightarrow E_0 + B_1 b^{-2} = -\epsilon_r A_1$; $n B_n b^{-(n+1)} = -\epsilon_r n A_n b^{n-1}$, $n \neq 1$.

Solving: $A_1 = -\frac{2E_0}{\epsilon_r+1}$, $B_1 = \frac{\epsilon_r-1}{\epsilon_r+1} b^2 E_0$.

$A_n = B_n = 0$ for $n \neq 1$.

$V_2(r,\phi) = -\left(1 - \frac{\epsilon_r-1}{\epsilon_r+1}\frac{b^2}{r^2}\right) E_0 r\cos\phi$.

$V_1(r,\phi) = -\frac{2}{\epsilon_r+1} E_0 r\cos\phi$.

$\bar{E} = -\bar{\nabla}V = -\bar{a}_r \frac{\partial V}{\partial r} - \bar{a}_\phi \frac{\partial V}{r\partial\phi}$.

$\bar{E}_2 = \bar{a}_x E_0 - \frac{\epsilon_r-1}{\epsilon_r+1}\left(\frac{b}{r}\right)^2 E_0 (\bar{a}_r\cos\phi + \bar{a}_\phi \sin\phi)$.

$\bar{E}_1 = \frac{2}{\epsilon_r+1} \bar{a}_x E_0 = \frac{2}{\epsilon_r+1} (\bar{a}_r\cos\phi - \bar{a}_\phi \sin\phi)$.

P.4-21 Starting from Eq. (4-134) and applying the boundary condition $V(b,\theta) = V_0$:

$V_0 = \frac{B_0}{b} + \left(\frac{B_1}{b^2} - E_0 b\right)\cos\theta - \sum_{n=2}^{\infty} B_n b^{-(n+1)} P_n(\cos\theta)$, $R \geq b$.

$\longrightarrow B_0 = bV_0$; $B_1 = E_0 b^3$; $B_n = 0$ for $n \geq 2$.

$\therefore V(R,\theta) = \frac{b}{R} V_0 - E_0\left[1 - \left(\frac{b}{R}\right)^2\right] R\cos\theta$, $R \geq b$.

$\bar{E}(R,\theta) = \bar{a}_R \left\{\frac{bV_0}{R^2} + E_0\left[1 + 2\left(\frac{b}{R}\right)^2\right]\cos\theta\right\}$

$- \bar{a}_\theta E_0 \left[1 - \left(\frac{b}{R}\right)^2\right] R\sin\theta$, $R \geq b$.

$\rho_s(\theta) = \epsilon_0 E_n\Big|_{R=b} = \epsilon_0 \frac{V_0}{b} + 3\epsilon_0 E_0 \cos\theta$.

P.4-22 $V_1(R,\theta) = \sum_{n=0}^{\infty} A_n R^n P_n(\cos\theta)$, $R \leq b$.

$V_2(R,\theta) = \sum_{n=0}^{\infty} \left(B_n R^n + C_n R^{-(n+1)}\right) P_n(\cos\theta)$, $R \geq b$.

For $R >> b$, $V_2(R,\theta) = -E_0 z = -E_0 R\cos\theta \longrightarrow B_1 = -E_0$; $B_n = C_n = 0$ for $n \neq 1$.

$\therefore V_2(R,\theta) = -E_0 R\cos\theta + C_1 R^{-2}\cos\theta$.

B.C.: $V_1(b,\theta) = V_2(b,\theta) \longrightarrow A_1 b = -E_0 b + C_1 b^{-2}$

$\epsilon_r \frac{\partial V_1}{\partial R}\Big|_{R=b} = \frac{\partial V_2}{\partial R}\Big|_{R=b} \longrightarrow \epsilon_r A_1 = -E_0 - 2C_1 b^{-3}$

$\Rightarrow A_1 = -\frac{3E_0}{\epsilon_r+2}$, $C_1 = \frac{\epsilon_r-1}{\epsilon_r+2} E_0 b^3$

$\therefore V_1(R,\theta) = -\frac{3E_0}{\epsilon_r+2} R\cos\theta$, $V_2(R,\theta) = -E_0 R\cos\theta + \frac{(\epsilon_r-1)b^3}{(\epsilon_r+2)R^2} E_0\cos\theta$.

$\bar{E}_1(R,\theta) = -\bar{\nabla}V_1 = \frac{3E_0}{\epsilon_r+2}(\bar{a}_R\cos\theta - \bar{a}_\theta\sin\theta) = \bar{a}_x \frac{3E_0}{\epsilon_r+2}$.

$\bar{E}_2(R,\theta) = -\bar{\nabla}V_2 = \bar{a}_R\left[1 + \frac{2(\epsilon_r-1)b^3}{(\epsilon_r+2)R^3}\right] E_0\cos\theta - \bar{a}_\theta\left[1 - \frac{(\epsilon_r-1)b^3}{(\epsilon_r+2)R^3}\right] E_0\sin\theta$.

Chapter 5

P.5-1 R_1 = Resistance per unit length of core $= \frac{1}{\sigma S_1} = \frac{1}{\sigma \pi a^2}$

R_2 = Resistance per unit length of coating $= \frac{1}{0.1\sigma S_2}$

Let b = Thickness of coating.

$S_2 = \pi(a+b)^2 - \pi a^2 = \pi(2ab+b^2)$.

a) $R_1 = R_2 \longrightarrow b = (\sqrt{11}-1)a = 2.32a$.

b) $I_1 = I_2 = I/2$: $J_1 = \frac{I}{2\pi a^2}$, $E_1 = \frac{J_1}{\sigma} = \frac{I}{2\pi a^2 \sigma}$,

$J_2 = \frac{I}{2S_2} = \frac{I}{20\pi a^2}$, $E_2 = \frac{J_2}{0.1\sigma} = \frac{I}{2\pi a^2 \sigma} = E_1$.

P.5-4 $I_1 = 0.7$ (A), $P_{d1} = 0.163$ (W); $I_2 = 0.140$ (A), $P_{d2} = 0.392$ (W);

$I_3 = 0.093$ (A), $P_{d3} = 0.261$ (W); $I_4 = 0.233$ (A), $P_{d4} = 0.436$ (W);

$I_5 = 0.467$ (A), $P_{d5} = 2.178$ (W).

P.5-5 $\rho = \rho_0 e^{-(r/a)^2}$, $\rho_0 = \frac{Q_0}{(4\pi/3)b^3} = \frac{10^{-1}}{(4\pi/3)(0.1)^3} = 0.239$ (C/m³).

a) $R < b$, $\bar{E}_1 = \bar{a}_R \frac{(4\pi/3)R^3\rho}{4\pi\epsilon R^2} = \bar{a}_R \frac{\rho_0 R}{3\epsilon} e^{-(R/a)^2}$

$= \bar{a}_R\, 7.5\times10^9 R\, e^{-9.43\times10^{11}t}$ (V/m);

$R > b$, $\bar{E}_2 = \bar{a}_R \frac{Q_0}{4\pi\epsilon_0 R^2} = \bar{a}_R \frac{9}{R^2}\times10^6$ (V/m).

b) $R < b$, $\bar{J}_1 = \sigma\bar{E}_1 = \bar{a}_R\, 7.5\times10^{10} R\, e^{-9.43\times10^{11}t}$ (A/m²);

$R > b$, $\bar{J}_2 = 0$.

P.5-6 a) $e^{-(t/\epsilon)t} = \frac{\rho}{\rho_0} = 0.01 \longrightarrow t = \frac{\ln 100}{(\sigma/\epsilon)} = 4.38\times10^{-12}$ (s)

$= 4.38$ (ps).

b) $W_i = \frac{\epsilon}{2}\int_{V'} E_i^2\, dv' = \frac{2\pi\rho_0^2 b^5}{45\epsilon} e^{-2(t/\epsilon)t} = (W_i)_0 [e^{-(t/\epsilon)t}]^2$.

$\therefore \frac{W_i}{(W_i)_0} = [e^{-(t/\epsilon)t}]^2 = 0.01^2 = 10^{-4}$. Energy dissipated as heat loss.

c) Electrostatic energy stored outside the sphere

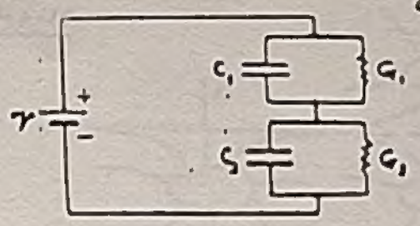
$W_e = \frac{\epsilon}{2}\int_b^\infty E_e^2\, 4\pi R^2 dR = \frac{Q_0^2}{8\pi\epsilon_0 b} = 45$ (kJ). Constant.

P.5-7 a) $R = \frac{l}{\sigma S} = \frac{V}{I} \longrightarrow \sigma = \frac{lI}{SV} = 3.537\times10^7$ (S/m).

b) $E = \frac{V}{l} = 6\times10^{-3}$ (V/m), or $E = \frac{J}{\sigma} = \frac{I}{\sigma S}$.

c) $P = VI = 1$ (W).

P.5-8



a) $C_1 = \frac{\epsilon_1 S}{d_1}$, $G_1 = \frac{\sigma_1 S}{d_1}$;

$C_2 = \frac{\epsilon_2 S}{d_2}$, $G_2 = \frac{\sigma_2 S}{d_2}$.

b) $P = V^2 G = V^2 \frac{G_1 G_2}{G_1 + G_2}$

$= V^2 S \frac{\sigma_1\sigma_2}{\sigma_1 d_1 + \sigma_2 d_2}$.

P.5-9 a) $G_1 = \frac{2\pi\sigma_1}{\ln(c/a)}$, $G_2 = \frac{2\pi\sigma_2}{\ln(b/c)}$.

$I = VG = V\frac{G_1 G_2}{G_1 + G_2} = \frac{2\pi\sigma_1\sigma_2 V}{\sigma_1\ln(b/c) + \sigma_2\ln(c/a)}$.

$J_1 = J_2 = \frac{I}{2\pi rL} = \frac{\sigma_1\sigma_2 V}{rL[\sigma_1\ln(b/c) + \sigma_2\ln(c/a)]}$.

b) $\rho_{sa} = \epsilon_1 E_1\big|_{r=a} = \frac{\epsilon_1\sigma_2 V}{aL[\sigma_1\ln(b/a) + \sigma_2\ln(c/a)]}$.

$\rho_{sb} = -\epsilon_2 E_2\big|_{r=b} = -\frac{\epsilon_2\sigma_2 V}{bL[\sigma_1\ln(b/c) + \sigma_2\ln(c/a)]}$.

$\rho_{sc} = -(\epsilon_2 E_2 - \epsilon_1 E_1)\big|_{r=c} = -\frac{(\epsilon_2\sigma_2 - \epsilon_1\sigma_1)V}{cL[\sigma_1\ln(b/c) + \sigma_2\ln(c/a)]}$.

P.5-10 $\bar{\nabla}^2 V = 0 \longrightarrow \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) = 0$.

$V(r) = c_1 \ln r + c_2$; B.C.: $V(a) = V_0$, $V(b) = 0$.

$V(r) = V_0 \frac{\ln(b/r)}{\ln(b/a)}$.

$\bar{E}(r) = -\bar{a}_r \frac{\partial V}{\partial r} = \bar{a}_r \frac{V_0}{r\ln(b/a)}$, $\bar{J} = \sigma\bar{E}$.

$I = \int_S \bar{J}\cdot d\bar{s} = \int_0^{\pi/4} \bar{J}\cdot(\bar{a}_r h r\, d\phi) = \frac{\pi\sigma h V_0}{2\ln(b/a)}$.

$R = \frac{V_0}{I} = \frac{2\ln(b/a)}{\pi\sigma h}$.

P.5-11 Assume a current I between the spherical surfaces.

$\bar{J} = \bar{a}_R \frac{I}{4\pi R^2} = \sigma\bar{E}$.

$V_0 = -\int_{R_1}^{R_2} \bar{E}\cdot d\bar{R} = \int_{R_1}^{R_2} \frac{I\, dR}{4\pi R^2\sigma} = \frac{I}{4\pi\sigma_0}\int_{R_1}^{R_2} \frac{dR}{R^2(1+k/R)}$

$= \frac{I}{4\pi\sigma_0}\int_{R_1}^{R_2} \frac{1}{k}\left(\frac{1}{R} - \frac{1}{R+k}\right) dR = \frac{I}{4\pi\sigma_0 k}\ln\frac{R_2(R_1+k)}{R_1(R_2+k)}$.

$R = \frac{V_0}{I} = \frac{1}{4\pi\sigma_0 k}\ln\frac{R_2(R_1+k)}{R_1(R_2+k)}$.

P.S-12 Assume I. $\bar{J}(R) = \bar{a}_R \frac{I}{S(R)}$.

$$S(R) = \int_0^{2\pi}\int_0^{\theta_0} R^2 \sin\theta \, d\theta \, d\phi = 2\pi R^2 (1-\cos\theta_0).$$

$$\bar{E}(R) = \frac{\bar{J}(R)}{\sigma} = \bar{a}_R \frac{I}{2\pi\sigma R^2(1-\cos\theta_0)}.$$

$$V_0 = -\int_{R_1}^{R_2} \bar{E}(R)\, dR = \frac{I(R_2-R_1)}{2\pi\sigma R_1 R_2 (1-\cos\theta_0)}.$$

$$\therefore R = \frac{V_0}{I} = \frac{R_2-R_1}{2\pi\sigma R_1 R_2(1-\cos\theta_0)}.$$

P.S-13 $\bar{\nabla}\cdot\bar{J} = \bar{\nabla}\cdot(\sigma\bar{E}) = \sigma\bar{\nabla}\cdot\bar{E} + (\bar{\nabla}\sigma)\cdot\bar{E} = 0.$

$\bar{E} = \bar{a}_R E$, $\bar{\nabla}\cdot\bar{E} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 E)$; $\bar{\nabla}\sigma = \bar{a}_R \frac{\partial\sigma}{\partial R} = -\bar{a}_R \frac{\sigma_1 R_1}{R^2}$.

$$\longrightarrow R\frac{\partial E}{\partial R} = -E, \quad \bar{E} = \bar{a}_R \frac{c}{R}.$$

$$V_0 = -\int_{R_1}^{R_2} \bar{E}\cdot d\bar{R} = -\int_{R_1}^{R_2} \frac{c}{R} dR = c \ln\frac{R_2}{R_1}.$$

$$c = \frac{V_0}{\ln(R_2/R_1)}, \quad \bar{E} = \bar{a}_R \frac{V_0}{R\ln(R_2/R_1)}.$$

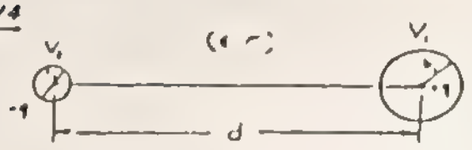
$$I = \int_S \bar{J}\cdot d\bar{s} = \int_S \sigma\bar{E}\cdot d\bar{s}$$

$$= \int_0^{2\pi}\int_0^{\theta_0} \left(\frac{\sigma_1 R_1}{R}\right)\left[\frac{V_0}{R\ln(R_2/R_1)}\right] R^2 \sin\theta \, d\theta \, d\phi$$

$$= \frac{2\pi\sigma_0 R_1 V_0 (1-\cos\theta_0)}{\ln(R_2/R_1)}.$$

$$\therefore R = \frac{V_0}{I} = \frac{\ln(R_2/R_1)}{2\pi\sigma_0 R_1(1-\cos\theta_0)}.$$

P.S-14



Assume charges +q and -q to concentrate at the centers of spheres 1 and 2 respectively.

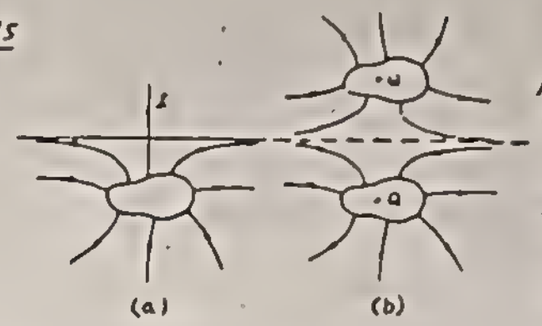
$d \gg b_1$, $d \gg b_2$. $V_1 \simeq \frac{q}{4\pi\epsilon}\left(\frac{1}{b_1} - \frac{1}{d-b_1}\right)$.

$$V_2 \simeq \frac{q}{4\pi\epsilon}\left(\frac{1}{d-b_2} - \frac{1}{b_2}\right).$$

$$C = \frac{q}{V_1 - V_2} = \frac{4\pi\epsilon}{\frac{1}{b_1}+\frac{1}{b_2}-\frac{1}{d-b_1}-\frac{1}{d-b_2}} = G\frac{\epsilon}{\sigma} = \frac{\epsilon}{R\sigma},$$

$$R = \frac{1}{4\pi\sigma}\left(\frac{1}{b_1}+\frac{1}{b_2}-\frac{1}{d-b_1}-\frac{1}{d-b_2}\right) \simeq \frac{1}{4\pi\sigma}\left(\frac{1}{b_1}+\frac{1}{b_2}-\frac{2}{d}\right).$$

P.S-15



The current flow pattern of the lower half of Fig. (b) if both the conductor and its image are fed with the same current is exactly the same as that of Fig. (a). All boundary conditions are satisfied.

The streamlines are similar to the electric field lines of a conductor and its image, both carrying a charge +Q, in the electrostatic case.

P.S-16 According to P.S-15, the current flow pattern would be the same as that of a whole sphere in unbounded earth medium. Hence the current lines are radial. Assume a current I.

$$\bar{J} = \bar{a}_R \frac{I}{2\pi R^2}, \quad \bar{E} = \bar{a}_R \frac{I}{2\pi\sigma R^2}.$$

$$V_0 = -\int_{\infty}^{b} E\, dR = -\frac{I}{2\pi\sigma}\int_{\infty}^{b} \frac{dR}{R^2} = \frac{I}{2\pi\sigma b}.$$

$$R = \frac{V_0}{I} = \frac{1}{2\pi\sigma b} = \frac{1}{2\pi(10^{-4})(25\times10^{-1})} = 6.36\times10^6 (\Omega).$$

P.S-17 The bounday conditions at y = 0 and y = b require that $Y(y) \sim \cos\left(\frac{n\pi}{b}y\right)$; the boundary condition at x=0 indicates that $X(x) \sim \sinh\left(\frac{n\pi}{b}x\right)$. Thus,

a) $$V(x,y) = \sum_{n=0}^{\infty} C_n \sinh\left(\frac{n\pi}{b}x\right)\cos\left(\frac{n\pi}{b}y\right).$$

B.C. at $x=a$: $$V(a,y) - V_0 = \sum_{n=0}^{\infty} C_n \sinh\left(\frac{n\pi}{b}a\right)\cos\left(\frac{n\pi}{b}y\right) = \sum_{n=0}^{\infty} B_n \cos\left(\frac{n\pi}{b}y\right).$$

$\int_0^b [\quad]\cos\left(\frac{n\pi}{b}y\right)dy$: $0 = B_n\left(\frac{b}{2}\right) \longrightarrow B_n = 0$ for $n \neq 0$.

For n=0, $V_0 = B_0 \longrightarrow C_0 = \frac{V_0}{\sinh(n\pi a/b)}$.

$$V(x,y) = V_0\left[\frac{\sinh(n\pi x/b)}{\sinh(n\pi a/b)}\cos\left(\frac{n\pi}{b}y\right)\right]_{n=0} = \frac{V_0}{a}x.$$

b) $\bar{J} = \sigma\bar{E} = -\sigma\bar{\nabla}V = -\bar{a}_x \frac{\sigma V_0}{a}$.

P5-18 $V(r,\phi) = \sum_{n=0}^{\infty} (A_n r^n + B_n r^{-n})(C_n \cos n\phi + D_n \sin n\phi)$.

B.C.: $V(r,\phi) = V(r,-\phi) \longrightarrow D_n = 0$.

$r \to \infty$, $V(r,\phi) = -\frac{1}{\sigma} J_0 r\cos\phi \longrightarrow A_n = C_n = 0$ for $n \neq 1$.

Write $V(r,\phi) = (K_1 r + \frac{K_2}{r})\cos\phi$. $K_1 = A_1 C_1$, $K_2 = B_1 C_1$.

$K_1 = -\frac{J_0}{\sigma}$

B.C.: $\left.\frac{\partial V}{\partial r}\right|_{r=b} = 0 \longrightarrow K_1 - \frac{K_2}{b^2} = 0$, $K_2 = b^2 K_1 = -\frac{J_0}{\sigma} b^2$.

$\therefore V(r,\phi) = -\frac{J_0}{\sigma}(r + \frac{b^2}{r})\cos\phi$.

$\bar{J} = -\sigma\bar{\nabla}V = -\sigma\left(\bar{a}_r \frac{\partial V}{\partial r} + \bar{a}_\phi \frac{\partial V}{r\partial\phi}\right)$

$= \bar{a}_r J_0 (1 - \frac{b^2}{r^2})\cos\phi - \bar{a}_\phi J_0 (1 + \frac{b^2}{r^2})\sin\phi$

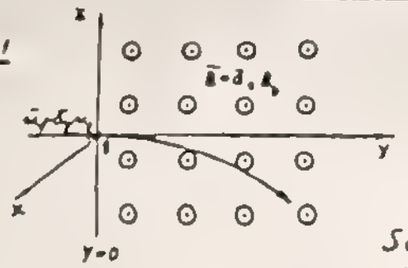
$= J_0(\bar{a}_r\cos\phi - \bar{a}_\phi\sin\phi) - \frac{J_0 b^2}{r^2}(\bar{a}_r\cos\phi + \bar{a}_\phi\sin\phi)$

$= \bar{a}_x J_0 - \frac{J_0 b^2}{r^2}(\bar{a}_r\cos\phi + \bar{a}_\phi\sin\phi)$, $r > a$.

$\bar{J} = 0$, $r < b$.

Chapter 6

P 6-1



$\frac{dv_x}{dt} = \frac{qB_0}{m} v_y = \omega_0 v_y$ ①

$\frac{dv_y}{dt} = -\frac{qB_0}{m} v_x = -\omega_0 v_x$ ② $\omega_0 = \frac{qB_0}{m}$

Combining ① and ②:

$\frac{d^2 v_x}{dt^2} + \omega_0^2 v_x = 0$.

Solution: $v_x = A\cos\omega_0 t + B\sin\omega_0 t$.

At $t=0$, $v_x = 0 \longrightarrow A = 0$.

$v_x = B\sin\omega_0 t$

Substituting v_x in ①: $v_y = -B\cos\omega_0 t$.

At $t=0$, $v_y = v_0 \longrightarrow B = -v_0$.

$\therefore v_y = v_0\cos\omega_0 t \longrightarrow y = \frac{v_0}{\omega_0}\sin\omega_0 t$ $(t=0, y=0)$.

$v_x = -v_0\sin\omega_0 t \longrightarrow z = \frac{v_0}{\omega_0}\cos\omega_0 t + C$ $(t=0, z=0 \longrightarrow C = -\frac{v_0}{\omega_0})$

$= -\frac{v_0}{\omega_0}(1 - \cos\omega_0 t)$.

We obtain $y^2 + (z + \frac{v_0}{\omega_0})^2 = (\frac{v_0}{\omega_0})^2$ --- Eq. of a shifted circle

P 6-2 $\frac{d\bar{v}}{dt} = -\frac{|q|}{m}(\bar{E} + \bar{v}\times\bar{B})$

a) $\bar{E} = \bar{a}_x E_0$, $\bar{B} = \bar{a}_z B_0$.

$\frac{dv_x}{dt} = 0$

$\frac{dv_y}{dt} = -\frac{|q|}{m} B_0 v_x$ $\longrightarrow$ $\begin{cases} v_x = 0 \\ v_y = (v_0 - \frac{E_0}{B_0})\cos\omega_0 t + \frac{E_0}{B_0} \\ v_z = (\frac{E_0}{B_0} - v_0)\sin\omega_0 t, \end{cases}$

$\frac{dv_z}{dt} = -\frac{|q|}{m}(E_0 - B_0 v_y)$

If the electron is injected at the origin ($x=y=z=0$) at $t=0$, we have $x=0$, $y = \frac{c}{\omega_0}\sin\omega_0 t + \frac{E_0}{B_0} t$, $z = -\frac{c}{\omega_0}(1-\cos\omega_0 t)$.

$\omega_0 = \frac{|q|}{m} B_0$

Equation of motion: $c = v_0 - \frac{E_0}{B_0}$.

$(y - \frac{E_0}{B_0} t)^2 + (z + \frac{c}{\omega_0})^2 = (\frac{c}{\omega_0})^2$.

If $E_0/B_0 = v_0$, $v_x = v_z = 0$, $v_y = v_0$; $x = z = 0$, $y = v_0 t$.

b) $\bar{E} = -\bar{a}_x E_0$, $\bar{B} = -\bar{a}_z B_0$.

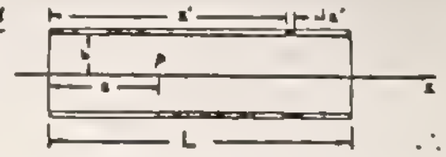
$\frac{dv_x}{dt} = \frac{|q|}{m} B_0 v_y = \omega_0 v_y$ } Circular motion (See P 6-4)

$\frac{dv_y}{dt} = -\omega_0 v_x$ } Circular motion (See P 6-4)

$\frac{dv_z}{dt} = \frac{|q|}{m} E_0$. } Helical motion with constant acceleration $|q|E_0/m$ in z direction

P 6-3 Straightforward application of Ampere's circuital law.

P 6-4



From Example 6-6, Eq (6-31),

$dB = \frac{\mu_0 I b^2}{2[(z'-z)^2 + b^2]^{3/2}}(\frac{N}{L}dz')$.

$\therefore B = \frac{\mu_0 N I b^2}{2L}\int_0^L \frac{dz'}{[(z'-z)^2 + b^2]^{3/2}}$.

or, $B = \frac{\mu_0 N I}{2L}\left[\frac{L-z}{\sqrt{(L-z)^2 + b^2}} + \frac{z}{\sqrt{z^2 + b^2}}\right] \longrightarrow \mu_0(\frac{N}{L})I$ as $L \to \infty$.

Direction of B is determined by the right-hand rule.

P 6-5 Eq. (6-22): $\bar{A} = \frac{\mu_0}{4\pi}\int_{v'} \frac{\bar{J}}{R} dv'$

$\bar{B} = \bar{\nabla}\times\bar{A} = \frac{\mu_0}{4\pi}\int_{v'} \bar{\nabla}\times(\frac{1}{R}\bar{J})\,dv' = \frac{\mu_0}{4\pi}\int_{v'}\left[\overbrace{\frac{1}{R}\bar{\nabla}\cdot\bar{J} + (\bar{\nabla}\frac{1}{R})\cdot\bar{J}}^{\text{From P. 2-26}}\right]dv'$

$\bar{\nabla}\cdot\bar{J} = 0$ because the curl operation $\bar{\nabla}\times$ is with respect to unprimed coordinates of the field point and $\bar{J}$ is a function of primed (source) coordinates

$\bar\nabla \frac{1}{R} = -\bar a_R \frac{1}{R^2}$.

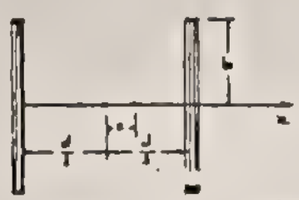
$\therefore \bar B = \frac{\mu_0}{4\pi}\int_{v'} \frac{-\bar a_R \times \bar J}{R^2}\,dv' = \frac{\mu_0}{4\pi}\int_{v'} \frac{\bar J \times \bar a_R}{R^2}\,dv'$.

$\bar\nabla \cdot \bar B = \frac{\mu_0}{4\pi}\int_{v'} \bar\nabla \cdot \left[\bar\nabla\left(\frac{1}{R}\right) \times \bar J\right] dv'$

$= \frac{\mu_0}{4\pi}\int_{v'} \left[\bar J \cdot \underbrace{\left(\bar\nabla \times \bar\nabla \frac{1}{R}\right)}_{0,\ \text{from Eq. (3-13)}} - \left(\bar\nabla \frac{1}{R}\right)\cdot \underbrace{\bar\nabla \times \bar J}_{0}\right] dv'$; from P. 3-13.

$= 0$.

P 6-6



Use Eq. (6-38).

$B_x = \frac{N\mu_0 I b}{2}\left\{\frac{1}{[(\frac{d}{2}+x)^2+b^2]^{3/2}} + \frac{1}{[(\frac{d}{2}-x)^2+b^2]^{3/2}}\right\}$.

a) At $x=0$, $B_x = \frac{N\mu_0 I b}{[(d/2)^2+b^2]^{3/2}}$.

b) $\frac{dB_x}{dx} = \frac{N\mu_0 I b}{2}\left\{-\frac{3}{2}\frac{2(\frac{d}{2}+x)}{[(\frac{d}{2}+x)^2+b^2]^{5/2}} + \frac{3}{2}\frac{2(\frac{d}{2}-x)}{[(\frac{d}{2}-x)^2+b^2]^{5/2}}\right\}$.

At the midpoint, $x=0$, $\frac{dB_x}{dx} = 0$.

c) $\frac{d^2B_x}{dx^2} = -\frac{3N\mu_0 I b}{2}\left\{\frac{1}{[(\frac{d}{2}+x)^2+b^2]^{5/2}} - \frac{5(\frac{d}{2}+x)^2}{[(\frac{d}{2}+x)^2+b^2]^{7/2}} + \frac{1}{[(\frac{d}{2}-x)^2+b^2]^{5/2}} - \frac{5(\frac{d}{2}-x)^2}{[(\frac{d}{2}-x)^2+b^2]^{7/2}}\right\}$.

At $x=0$, $\frac{d^2B_x}{dx^2} = -\frac{6N\mu_0 I b}{2}\left\{\frac{(d/2)^2+b^2-5(d/2)^2}{[(d/2)^2+b^2]^{7/2}}\right\}$,

which vanishes if $b^2-4(d/2)^2=0$, or $b=d$.

P 6-7

Use Eq. (6-35) for a wire of length 2L. $\bar B = \bar a_\phi \frac{\mu_0 I L}{2\pi r\sqrt{L^2+r^2}}$.

$\alpha = \frac{2\pi}{2N} = \frac{\pi}{N}$. In this problem, $\bar B = \bar a_n N\left(\frac{\mu_0 I L}{2\pi r b}\right) = \bar a_n \frac{\mu_0 N I}{2\pi b}\tan\frac{\pi}{N}$.

$\frac{L}{r} = \tan\alpha = \tan\frac{\pi}{N}$.

When N is very large, $\tan\frac{\pi}{N} \to \frac{\pi}{N}$, $\bar B \to \bar a_n \frac{\mu_0 I}{2b}$,

which is the same as Eq. (6-35) with $z=0$.

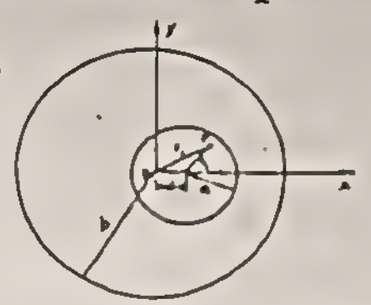
P 6-8 $B_\phi = \frac{\mu_0 N I}{2\pi r}$.

$\Psi = \int_S B_\phi\,ds = \frac{\mu_0 N I}{2\pi}\int_a^b \frac{1}{r} h\,dr = \frac{\mu_0 N I h}{2\pi}\ln\frac{b}{a}$.

If B_ϕ at $r = \frac{a+b}{2}$ is used, $\Psi' = \frac{\mu_0 N I h}{\pi}\left(\frac{b-a}{b+a}\right)$.

% error $= \frac{\Psi'-\Psi}{\Psi}\times 100\% = \left[\frac{2(b-a)}{(b+a)\ln(b/a)} - 1\right]\times 100\%$.

P 6-9



$\bar J = \bar a_z J$, $\oint \bar B \cdot d\bar l = \mu_0 I$

If there were no hole, $2\pi r_1 B_{\phi 1} = \mu_0 \pi r_1^2 J$

$B_{\phi 1} = \frac{\mu_0 r_1}{2} J \begin{cases} B_{x1} = -\frac{\mu_0 J}{2} y_1 \\ B_{y1} = +\frac{\mu_0 J}{2} x_1 \end{cases}$.

For $-J$ in the hole portion:

$B_{\phi 2} = -\frac{\mu_0 r_2}{2} J \begin{cases} B_{x1} = -\frac{\mu_0 J}{2} y_2 \\ B_{y1} = -\frac{\mu_0 J}{2} x_2 \end{cases}$.

Superposing $B_{\phi 1}$ and $B_{\phi 2}$ and noting that $y_1 = y_2$ and $x_1 = x_2 + d$, we have

$B_x = B_{x1} + B_{x2} = 0$ and $B_y = B_{y1} + B_{y2} = \frac{\mu_0 J}{2} d$.

P 6-11 $\bar B = \bar\nabla \times \bar A$, $\bar B = \bar a_\phi B = \bar a_\phi\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) = -\bar a_\phi \frac{\partial A_z}{\partial r}$.

For $0 \le r \le b$, Eq. (6-10) gives $\bar B_1 = \bar a_\phi \frac{\mu_0 I}{2\pi b^2} r$.

For $r \ge b$, Eq. (6-11) gives $\bar B_2 = \bar a_\phi \frac{\mu_0 I}{2\pi}\frac{1}{r}$.

Integrating, $\bar A_1 = \bar a_z\left[-\frac{\mu_0 I}{4\pi}\left(\frac{r}{b}\right)^2 + c_1\right]$, $0 \le r \le b$

$\bar A_2 = \bar a_z\left[-\frac{\mu_0 I}{2\pi}\ln r + c_2\right]$, $r \ge b$

At $r=b$, $\bar A_1 = \bar A_2$ ——— $c_2 = -\frac{\mu_0 I}{4\pi} + \frac{\mu_0 I}{2\pi}\ln b + c_1$.

$\therefore \bar A_1 = \bar a_z\left[-\frac{\mu_0 I}{4\pi}\left[\ln\left(\frac{r}{b}\right)^2 + 1\right] + c_1\right]$, $r \ge b$.

P 6-12 Eq. (6-38) for one wire: $\bar A = \bar a_z \frac{\mu_0 I}{2\pi}\ln\frac{\sqrt{L^2+r^2}+L}{\sqrt{L^2+r^2}-L}$.

For two wires with equal and opposite currents:

a) $\bar A = \bar a_z \frac{\mu_0 I}{4\pi}\ln\left[\frac{\sqrt{L^2+r_1^2}+L}{\sqrt{L^2+r_1^2}-L}\cdot\frac{\sqrt{L^2+r_2^2}-L}{\sqrt{L^2+r_2^2}+L}\right] = \bar a_z \frac{\mu_0 I}{2\pi}\ln\left[\frac{r_2}{r_1}\cdot\frac{\sqrt{L^2+r_1^2}+L}{\sqrt{L^2+r_2^2}+L}\right]$.

b) for a very long two-wire transmission line, $L \to \infty$:

$$\bar{A} = \bar{a}_z \frac{\mu_0 I}{2\pi} \ln\left(\frac{r_2}{r_1}\right) = \bar{a}_z \frac{\mu_0 I}{4\pi} \ln \frac{(\frac{d}{2}+y)^2 + x^2}{(\frac{d}{2}-y)^2 + x^2}.$$

c) $\bar{B} = \nabla \times \bar{A} = \bar{a}_x \frac{\partial A_z}{\partial y} - \bar{a}_y \frac{\partial A_z}{\partial x}$

$$= \bar{a}_x \frac{\mu_0 I}{2\pi}\left[\frac{\frac{d}{2}+y}{(\frac{d}{2}+y)^2+x^2} - \frac{\frac{d}{2}-y}{(\frac{d}{2}-y)^2+x^2}\right]$$

$$-\bar{a}_y \frac{\mu_0 I}{2\pi}\left[\frac{x}{(\frac{d}{2}+y)^2+x^2} - \frac{x}{(\frac{d}{2}-y)^2+x^2}\right] = \frac{\mu_0 I}{2\pi}\left[\bar{a}_{r_1}\frac{1}{r_1} - \bar{a}_{r_2}\frac{1}{r_2}\right].$$

P.6-14 Apply divergence theorem to $(\bar{F}\times\bar{C})$:

$$\int_v \nabla\cdot(\bar{F}\times\bar{C})\,dv = \oint_S (\bar{F}\times\bar{C})\cdot d\bar{s}.$$

Now $\nabla\cdot(\bar{F}\times\bar{C}) = \bar{C}\cdot(\nabla\times\bar{F}) - \bar{F}\cdot(\nabla\times\bar{C}) = \bar{C}\cdot(\nabla\times\bar{F})$

$(\bar{F}\times\bar{C})\cdot d\bar{s} = -(\bar{F}\times d\bar{s})\cdot\bar{C}$.

Thus, $\bar{C}\cdot\int_v(\nabla\times\bar{F})\,dv = -\bar{C}\cdot\oint_S \bar{F}\times d\bar{s}$

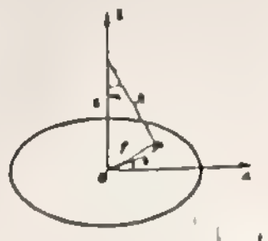
$\longrightarrow \int_v(\nabla\times\bar{F})\,dv = -\oint_S \bar{F}\times d\bar{s}$ because $\bar{C}$ is an arbitrary constant vector.

P.6-15 a) $r<a$: $\bar{H} = \bar{a}_z nI$, $\bar{B} = \bar{a}_z \mu nI$, $\bar{M} = \frac{\bar{B}}{\mu_0} - \bar{H} = \bar{a}_z(\frac{\mu}{\mu_0}-1)nI$ } Eq.(6-13)

$a<r<b$: $\bar{H} = \bar{a}_z nI$, $\bar{B} = \bar{a}_z \mu_0 nI$, $\bar{M} = 0$

b) $J_m = \nabla\times\bar{M} = 0$; $J_{ms} = \bar{M}\times\bar{a}_n = (\bar{a}_z\times\bar{a}_r)(\frac{\mu}{\mu_0}-1)nI = \bar{a}_\phi(\frac{\mu}{\mu_0}-1)nI$.

P.6-17



a) $V_m = \frac{I}{4\pi}\int \frac{d\bar{s}\cdot\bar{a}_R}{R^2} = \frac{I}{4\pi}\Omega$.

$d\bar{s}\cdot\bar{a}_R = (\cos\alpha)\rho\,d\rho\,d\phi = \frac{z}{\sqrt{z^2+\rho^2}}\rho\,d\rho\,d\phi$,

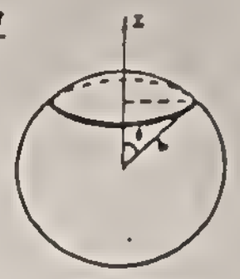
$R = \sqrt{z^2+\rho^2}$:

$$V_m = \frac{I}{4\pi}\int_0^{2\pi}\int_0^b \frac{z}{(z^2+\rho^2)^{3/2}}\rho\,d\rho\,d\phi = \frac{I}{2}\left(1 - \frac{z}{\sqrt{z^2+b^2}}\right).$$

b) $\bar{B} = -\mu_0\nabla V_m = -\bar{a}_z\mu_0\frac{\partial V_m}{\partial z} = \bar{a}_z\frac{\mu_0 I b^2}{2(z^2+b^2)^{3/2}}$,

which is the same as Eq.(6-38).

P.6-18



$\bar{M} = \bar{a}_z M_0$

a) $\bar{J}_m = \nabla\times\bar{M} = 0$.

$\bar{J}_{ms} = \bar{M}\times\bar{a}_n = \bar{a}_z M_0 \times \frac{1}{b}(\bar{a}_x x + \bar{a}_y y + \bar{a}_z z)$

$= -\frac{M_0}{b}(-\bar{a}_x y + \bar{a}_y x) = -\bar{a}_\phi \frac{M_0}{b}\sqrt{x^2+y^2}$

$= \bar{a}_\phi M_0 \sin\theta$.

[or, $\bar{J}_{ms} = (\bar{a}_x\cos\theta - \bar{a}_y\sin\theta)M_0\times\bar{a}_r$

$= \bar{a}_\phi M_0\sin\theta$]

b) Apply Eq.(6-38) to a loop of radius $b\sin\theta$ carrying a current $J_{ms}\,b\,d\theta$:

$$d\bar{B} = \bar{a}_z\frac{\mu_0(J_{ms}\,b\,d\theta)(b\sin\theta)^2}{2(b)^3} = \bar{a}_z\frac{\mu_0 M_0}{2}\sin^3\theta\,d\theta$$

$$\bar{B} = \int d\bar{B} = \bar{a}_z\frac{\mu_0 M_0}{2}\int_0^\pi \sin^3\theta\,d\theta = \bar{a}_z\frac{4}{3}\mu_0 M_0 = \frac{4}{3}\mu_0\bar{M}.$$

P.6-19 a) $\alpha_g = \frac{1}{\mu_0 S} = \frac{1}{4\pi\times10^{-7}\times\pi\times0.025^2} = 1.21\times10^5\ (H^{-1})$

$$\alpha_c = \frac{2\pi\times0.05 - 0.001}{2\pi\times10^3(4\pi\times10^{-7})\times\pi\times0.025^2} = 6.75\times10^4\ (H^{-1}).$$

b) $\bar{B}_g = \bar{B}_c = \bar{a}_\phi\frac{10^{-4}}{\pi\times0.025^2} = \bar{a}_\phi\,5.09\times10^{-3}\ (T)$

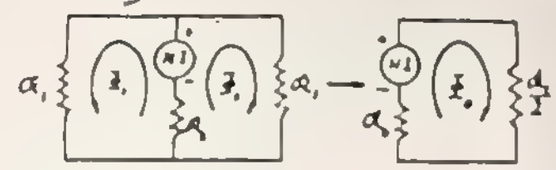
$\bar{H}_g = \frac{1}{\mu_0}\bar{B}_g = \bar{a}_\phi\frac{5.09\times10^{-3}}{4\pi\times10^{-7}} = \bar{a}_\phi\,4.05\times10^3\ (A/m)$

$\bar{H}_c = \frac{1}{\mu_0\mu_r}\bar{B}_c = \bar{a}_\phi\frac{4.05\times10^3}{3000} = \bar{a}_\phi\,1.35\ (A/m)$.

c) $NI = \Phi(\alpha_c + \alpha_g)$, $I = \frac{1}{N}\Phi(\alpha_c+\alpha_g) = \frac{10^{-4}}{500}\times2.3333\times10^4$

$= 0.0256\ (A) = 25.6\ (mA)$.

P.6-20 Magnetic circuit:



$S = 10^{-3}\ (m^2)$

$\frac{1}{\mu_0 S} = \frac{1}{(4\pi\times10^{-7})\times10^{-3}} = 795.8\ H^{-1}$.

Neglecting leakage flux and assuming constant flux density over S.

$\alpha_0 = \frac{1}{\mu_0 S}\left(0.002 - \frac{0.24-0.01}{3000}\right) = 1.60\times10^6\ (H^{-1})$.

$\alpha_1 = \frac{1}{\mu_0 S}\left(\frac{0.24+0.2}{3000}\right) = 0.102\times10^6\ (H^{-1})$.

$$\Phi_0 = \frac{NI}{a_0 + a_1/2} = \frac{200 \times 3}{(1.60 + \frac{0.01}{2}) \times 10^{-4}} = 3.63 \times 10^{-4} \text{ (Wb)}.$$

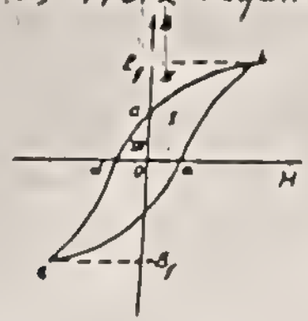
$$\Phi_1 = \frac{\Phi_0}{2} = 1.82 \times 10^{-4} \text{ (Wb)}.$$

b) $H_1 = \frac{B_1}{\mu_0 \mu_r} = \frac{\Phi_1}{\mu_0 \mu_r S} = (7.95 \times 10^5) \frac{1.82 \times 10^{-4}}{5000} = 28.9 \text{ (A/m)}$

$(H_0)_1 = \frac{B_1}{\mu_0} = \frac{1}{\mu_0 S} \Phi_0 = (7.95 \times 10^5) \times 3.63 \times 10^{-4} = 28.9 \times 10^3$ (A/m), in air gap.

$(H_0)_1 = (H_0)_1 / 5000 = 57.8$ (A/m), in core.

P.6-21 a) Work required per unit length in time dt:



$$P_i dt = nI\, d\Sigma.$$

Work per unit volume in dt:

$$dW = \frac{1}{S} P_i dt = nI\, dB = H dB.$$

Thus, $W_1 = \int_0^{B_f} H dB.$

b) Work done per unit volume in changing from 0 to B_f along path ab is W_1, which is represented by areas I and II.

Along path bc, B is decreased, inducing a voltage that tends to maintain the current. Work is done against the source. The work per unit volume W_2 is represented by -(area II). In going from c to d, the direction of current is reversed and the work done W_3 is represented by area III. Same amount of work is done in changing B along the path from d to e and back to a as that required in going from a to b through c to d.

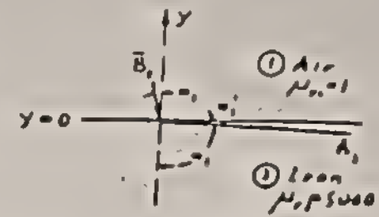
∴ Work done per unit volume in one cycle = $2(W_1 + W_2 + W_3)$
$= 2 \times \text{Area}[(\text{I} + \text{II}) - \text{II} + \text{III}] = \text{Area of the hysteresis loop.}$

P.6-23 $\bar{H}_1 = -\bar{\nabla} V_{m1}$, $\bar{H}_2 = -\bar{\nabla} V_{m2}$.

Boundary Conditions: $\mu_1 H_{1n} = \mu_2 H_{2n} \longrightarrow \mu_1 \frac{\partial V_{m1}}{\partial n} = \mu_2 \frac{\partial V_{m2}}{\partial n}$,

$H_{2t} - H_{1t} = J_{1n} \longrightarrow \frac{\partial V_{m2}}{\partial t} - \frac{\partial V_{m1}}{\partial t} = J_{1n}$.

P.6-24



a) $\bar{B}_1 = \bar{a}_x 0.5 - \bar{a}_y 10$ (mT)

$\bar{B}_2 = \bar{a}_x B_{2x} + \bar{a}_y B_{2y}$.

$H_{1x} = \frac{B_{1x}}{5000\mu_0} = H_{2x} = \frac{0.5}{\mu_0}$ ⟶ $B_{2x} = 2{,}500$ (mT).

$B_{2y} = B_{1y} = -10$ (mT)

$\tan\alpha_2 = \frac{\mu_2}{\mu_1} \tan\alpha_1$ ∴ $\bar{B}_2 = \bar{a}_x 2500 - \bar{a}_y 10$ (mT).

$= 5000 \tan 0.05 \approx 250$ ⟶ $\alpha_2 = 89.77°$, $\alpha_2' = 0.23°$.

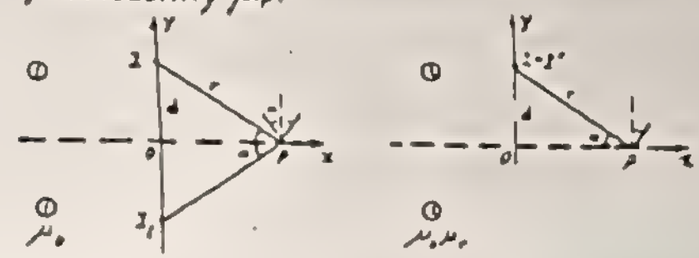
b) If $\bar{B}_2 = \bar{a}_x 10 + \bar{a}_y 0.5$ (mT), $\bar{B}_1 = \bar{a}_x B_{1x} + \bar{a}_y B_{1y}$.

$H_{1x} = \frac{B_{1x}}{\mu_1} = H_{2x} = \frac{B_{2x}}{\mu_2}$ ⟶ $B_{1x} = \frac{\mu_1}{\mu_2} B_{2x} = \frac{10}{5000} = 2$ mT

$B_{1y} = B_{2y} = 0.5$. ∴ $\bar{B}_1 = \bar{a}_x 0.002 + \bar{a}_y 0.5$ (mT).

$\alpha_1 = \tan^{-1} \frac{B_{1x}}{B_{1y}} \approx \frac{0.002}{0.5} = 0.004 \text{ (rad)} = 0.23°$.

P.6-25 a) Consider two situations: ① I and I_1 both in air; and ② I and -I both in magnetic medium with relative permeability μ_r.



Find B_{1y} and H_{1x} at P(y=0). Find B_{2y} and H_{2x} at P(y=0).

$B_{1y} = \frac{\mu_0}{2\pi r}(I + I_1)\cos\theta = \frac{\mu_0\mu_r}{\pi(\mu_r+1)} \frac{I}{r}\frac{d}{r}$; $B_{2y} = \frac{\mu_0\mu_r}{2\pi r}(I - I_1)\cos\theta = \frac{\mu_0\mu_r}{\pi(\mu_r+1)} \frac{I}{r}\frac{d}{r}$.

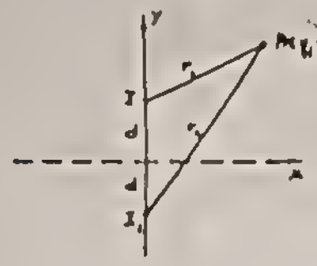
$B_{1x} = \frac{\mu_0}{2\pi r}(I - I_1)\sin\theta = \frac{\mu_0}{\pi(\mu_r+1)} \frac{d}{r^2} I$; $B_{2x} = \frac{\mu_0\mu_r}{2\pi r}(I - I_1)\sin\theta = \frac{\mu_0\mu_r}{\pi(\mu_r+1)} \frac{d}{r^2} I$.

$H_{1x} = \frac{B_{1x}}{\mu_0} = \frac{I}{\pi(\mu_r+1)} \frac{d}{r^2}$; $H_{2x} = \frac{B_{2x}}{\mu_0\mu_r} = \frac{I}{\pi(\mu_r+1)} \frac{d}{r^2}$.

∴ $B_{1y} = B_{2y}$ and $H_{1x} = H_{2x}$ (Boundary conditions satisfied)

b) For $\mu_r \gg 1$, $I_1 = \frac{\mu_r - 1}{\mu_r + 1} I \approx I$.

Refer to following figure.



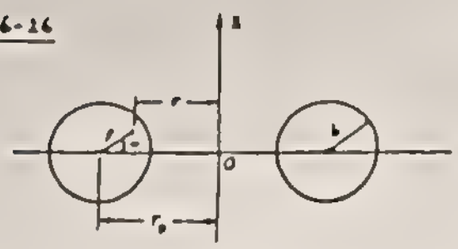
$$\bar{B}_1 = \frac{\mu_0 I}{2\pi r_1}\left(-\bar{a}_x \frac{y-d}{r_1} + \bar{a}_y \frac{x}{r_1}\right).$$

$$\bar{B}_{I_1} = \frac{\mu_0 I}{2\pi r_1}\left(-\bar{a}_x \frac{y+d}{r_1} + \bar{a}_y \frac{x}{r_1}\right).$$

$$\therefore \bar{B} = \bar{B}_1 + \bar{B}_{I_1}$$

$$= -\bar{a}_x \frac{\mu_0 I}{2\pi}\left[\frac{y-d}{(y-d)^2+x^2} + \frac{y+d}{(y+d)^2+x^2}\right] + \bar{a}_y \frac{\mu_0 I x}{2\pi}\left[\frac{1}{(y-d)^2+x^2} + \frac{1}{(y+d)^2+x^2}\right].$$

P.6-16



$$\bar{B} = \bar{a}_\phi B_\phi = \bar{a}_\phi \frac{\mu_0 NI}{2\pi r}.$$

$$r = r_0 - \rho\cos\alpha.$$

$$\Psi = \frac{\mu_0 NI}{2\pi}\int_0^b\int_0^{2\pi} \frac{\rho\, d\alpha\, d\rho}{r_0 - \rho\cos\alpha}$$

$$= \frac{\mu_0 NI}{2\pi}\int_0^b \left[\frac{2\pi}{\sqrt{r_0^2-\rho^2}}\right]\rho\, d\rho$$

$$= \mu_0 NI\left(r_0 - \sqrt{r_0^2-b^2}\right).$$

$$\therefore L = \frac{N\Psi}{I} = \mu_0 N^2\left(r_0 - \sqrt{r_0^2-b^2}\right).$$

If $r_0 \gg b$, $B_\phi \approx \frac{\mu_0 NI}{2\pi r_0}$ (constant).

$$\Psi \approx B_\phi S = \frac{\mu_0 NI}{2\pi r_0}\cdot \pi b^2 = \frac{\mu_0 N b^2 I}{2 r_0}.$$

$$L = \frac{N\Psi}{I} \approx \frac{\mu_0 N^2 b^2}{2 r_0}.$$

P.6-17 For $b < r \le (b+d)$, $\bar{B}_1 = \bar{a}_\phi B_{\phi 1} = \bar{a}_\phi \frac{\mu_0 I}{2\pi r}\left[1 - \frac{\pi(r^2-b^2)}{\pi(b+d)^2-\pi b^2}\right]$

$$= \bar{a}_\phi \frac{\mu_0 I}{2\pi r}\left[\frac{(b+d)^2-r^2}{(b+d)^2-b^2}\right].$$

Magnetic energy per unit length stored in the outer conductor

$$W_{m_1}' = \frac{1}{2\mu_0}\int_b^{b+d} B_{\phi 1}^2\, 2\pi r\, dr$$

$$= \frac{\mu_0 I^2}{4\pi}\left\{\frac{(b+d)^4}{[(b+d)^2-b^2]^2}\ln\left(1+\frac{d}{b}\right) + \frac{b^2-3(b+d)^2}{4[(b+d)^2-b^2]}\right\}.$$

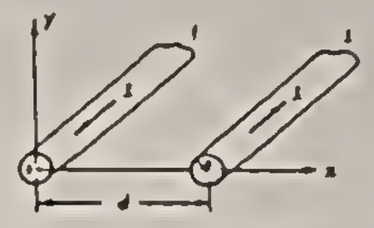
From Eqs. (6-154) - (6-155b) on p.246 we have

$$L' = \frac{2}{I^2}\left(W_{m_1}' + W_{m_2}' + W_{m_3}'\right)$$

$$= \frac{\mu_0}{2\pi}\left\{\frac{1}{4} + \ln\frac{b}{a} - \frac{(b+d)^4}{[(b+d)^2-b^2]^2}\ln\left(1+\frac{d}{b}\right) + \frac{b^2-3(b+d)^2}{4[(b+d)^2-b^2]}\right\}$$

(H/m).

P.6-28



$$\Psi_2' = \int_a^{d-a}(B_{y1} + B_{y2})\, dx$$

$$= \int_a^{d-a}\left[\frac{\mu_0 I}{2\pi x} + \frac{\mu_0 I}{2\pi(d-x)}\right]dx$$

$$= \frac{\mu_0 I}{\pi}\ln\left(\frac{d-a}{a}\right) = \frac{\mu_0 I}{\pi}\ln\frac{d}{a}.$$

$$\therefore L_2' = \frac{\Psi_2'}{I} = \frac{\mu_0}{\pi}\ln\frac{d}{a} \quad (H/m).$$

$$L' = L_1' + L_2' = \frac{\mu_0}{4\pi} + \frac{\mu_0}{\pi}\ln\frac{d}{a} \quad (H/m).$$

P.6-29 For a current I in the long straight wire,

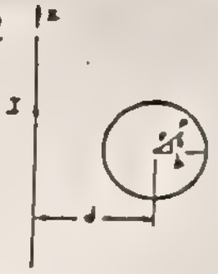
$$\bar{B} = \bar{a}_\phi \frac{\mu_0 I}{2\pi r}.$$

$$\Lambda_{11} = \int_S \bar{B}\cdot d\bar{s} = 2\int B_\phi \frac{b}{\sqrt{3}}(r-d)\, dr = \frac{\mu_0 I}{\pi\sqrt{3}}\int_d^{d+\frac{\sqrt{3}}{2}b}\left(\frac{r-b}{r}\right)dr$$

$$= \frac{\mu_0 I}{\pi\sqrt{3}}\left[\frac{\sqrt{3}}{2}b - d\ln\left(1+\frac{\sqrt{3}b}{2d}\right)\right].$$

$$L_{11} = \frac{\Lambda_{11}}{I} = \frac{\mu_0}{\pi}\left[\frac{b}{2} - \frac{d}{\sqrt{3}}\ln\left(1+\frac{\sqrt{3}b}{2d}\right)\right] \quad (H).$$

P.6-30



Assume a current I.

$\bar{B}$ at $P(r,\theta)$ is $\bar{a}_\phi \frac{\mu_0 I}{2\pi(d + r\cos\theta)}$.

$$\Lambda_{11} = \frac{\mu_0 I}{2\pi}\int_0^b\int_0^{2\pi}\frac{r\, dr\, d\theta}{d + r\cos\theta} = \frac{\mu_0 I}{2\pi}\int_0^b \frac{2\pi r\, dr}{\sqrt{d^2-r^2}}$$

$$= \mu_0 I\left(d - \sqrt{d^2-b^2}\right).$$

$$L_{11} = \mu_0\left(d - \sqrt{d^2-b^2}\right).$$

P.6-31 Since $h_1 \gg h_2$, the magnetic flux due to the long loop linking with the small loop can be approximated by that due to two infinitely long wires carrying equal and opposite current I.

$$\Lambda_{12} = \frac{\mu_0 I}{2\pi}\int_0^{w_2}\left(\frac{1}{d+x} - \frac{1}{w_1+d+x}\right)dx = \frac{\mu_0 I}{2\pi}\ln\left(\frac{w_2+d}{d}\cdot\frac{w_1+d}{w_1+w_2+d}\right).$$

$$L = \frac{\Lambda_{12}}{I} = \frac{\mu_0}{2\pi}\ln\frac{(w_2+d)(w_1+d)}{d(w_1+w_2+d)}.$$

P.6-32 Eq. (6-140): $W_2 = \frac{1}{2}L_1 I_1^2 + L_{12} I_1 I_2 + \frac{1}{2}L_2 I_2^2$.

a) $W_2 = \frac{I_1^2}{2}\left[L_1\left(\frac{I_2}{I_1}\right)^2 + 2L_{12}\left(\frac{I_2}{I_1}\right) + L_2\right]$

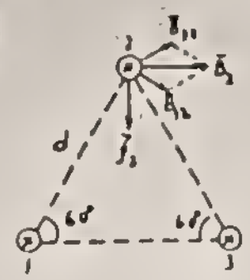
$$= \frac{I_1^2}{2}\left(L_1 x^2 + 2L_{12}x + L_2\right), \quad x = \frac{I_2}{I_1}.$$

$$\frac{dW_1}{dx} = \frac{I_1^2}{2}(2L_1 x + 2L_{12}) = 0, \qquad \frac{d^2W_1}{dx^2} = I_1^2 L_1 > 0.$$

$$\therefore \quad x = \frac{I_1}{I_2} = -\frac{L_{12}}{L_1} \text{ for minimum } W_1.$$

b) $(W_1)_{min} = \frac{I_1^2}{2}\left(L_1 - \frac{L_{12}^2}{L_2}\right) \geq 0 \longrightarrow L_{12} \leq \sqrt{L_1 L_2}$.

P.6-11



$I_1 = I_2 = I_3 = 25$ (A)

$d = 0.15$ (m)

$$\bar{B}_1 = \bar{a}_x 2B_{12}\cos 30° = \bar{a}_x 2\left(\frac{\mu_0 I}{2\pi d}\right)\frac{\sqrt{3}}{2}$$

$$= \bar{a}_x \frac{\mu_0 I\sqrt{3}}{2\pi d}.$$

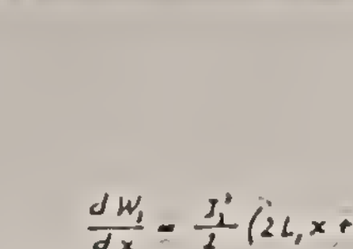
Force per unit length on wire 2:

$$\bar{f}_2 = -\bar{a}_y I B_1 = -\bar{a}_y \frac{\mu_0 I^2\sqrt{3}}{2\pi d}$$

$$= -\bar{a}_y 1150\mu_0 = -\bar{a}_y 1.44\times10^{-3} \text{ (N/m)}.$$

Forces on all three wires are of equal magnitude and toward the center of the triangle.

P.6-34



Elemental strip dy: $dI = \frac{I}{w}dy$,

$$|d\bar{H}| = \frac{dI}{2\pi r} = \frac{I\,dy}{2\pi w\sqrt{D^2+y^2}}.$$

Symmetry ⟶ $\bar{H}$ has only a y-component.

$$\bar{H} \text{ (at wire)} = \bar{a}_y \int dH\cdot\left(\frac{D}{r}\right)$$

$$= \bar{a}_y 2\int_0^{w/2} \frac{ID}{2\pi w\sqrt{D^2+y^2}}\,dy$$

$$= \bar{a}_y \frac{I}{\pi w}\tan^{-1}\left(\frac{w}{2D}\right).$$

$$\bar{f} = \bar{I}\times\bar{B}$$

$$= (-\bar{a}_z I)\times(\mu_0\bar{H}) = \bar{a}_x \frac{\mu_0 I^2}{\pi w}\tan^{-1}\left(\frac{w}{2D}\right) \quad \text{(N/m)}.$$

P.6-35 $\bar{B}$ due I_1 in straight wire in z direction at an elemental arc b dθ on the circular loop:

$$\bar{B} = \bar{a}_\phi \frac{\mu_0 I_1}{2\pi(d+b\cos\theta)}$$

Vertical component of $d\bar{F}$ on b dθ at θ is cancelled by that on b dθ at −θ

$$\therefore \bar{F}_{\text{on loop}} = -\bar{a}_x 2\int_0^\pi (I_2 b\,d\theta)B\cos\theta$$

$$\bar{F}_{\text{on loop}} = -\bar{a}_x \frac{\mu_0 I_1 I_2 b}{\pi}\int_0^\pi \frac{\cos\theta\,d\theta}{d+b\cos\theta}.$$

$$= a_x \mu_0 I_1 I_2\left[\frac{1}{\sqrt{1-(b/d)^2}} - 1\right] \quad \text{(Repulsive force)}.$$

P.6-36 Resolve the circular loop into many small loops, each with a magnetic dipole moment $d\bar{m} = I_2\,d\bar{s}$.

$$d\bar{T} = d\bar{m}\times\bar{B}.$$

$$\bar{T} = \int d\bar{T} = I_2\int d\bar{s}\times\bar{B}$$

$$= -\bar{a}_z I_2 \sin\alpha\int B\,ds = -\bar{a}_z \mu_0 I_1 I_2\left(d-\sqrt{d^2-b^2}\right)\sin\alpha$$

in the direction of aligning the direction of the flux by I_2 in the loop to that of $\bar{B}$ due to I_1 in the straight wire.

{ $\int B\,ds$ over the circular loop has been found in problem P.6-30 as Λ_{12}. }

P.6-37 Magnetic flux density at the center of the large circular turn of wire carrying current I_2 is

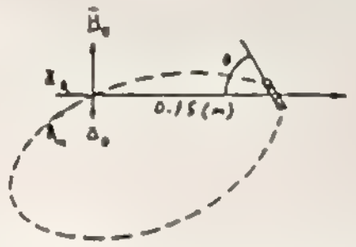
$$\bar{B}_2 = \bar{a}_{z2}\frac{\mu_0 I_2}{2r_2}. \quad \text{(Set z=0 in Eq.(6-38))}$$

Torque on the small circular wire:

$$\bar{T} = \bar{m}_1\times\bar{B}_2 = (\bar{a}_{z1} I_1\pi r_1^2)\times(\bar{a}_{z2}\frac{\mu_0 I_2}{2r_2}) = (\bar{a}_{z1}\times\bar{a}_{z2})\frac{\mu_0 I_1 I_2\pi r_1^2}{2r_2}$$

⟶ Magnitude $= \frac{\mu_0 I_1 I_2\pi r_1^2}{2r_2}\sin\theta$, in a direction to align the magnetic fluxes produced by I_1 and I_2.

P.6-38



$\bar{B}_m$ (magnetized compass needle)

$$= \frac{\mu_0 m}{4\pi R^3}(\bar{a}_R 2\cos\theta + \bar{a}_\theta\sin\theta)$$

$$= \frac{(4\pi\times10^{-7})\times2}{4\pi(0.15)^3}(\bar{a}_R 2\cos\theta + \bar{a}_\theta\sin\theta)$$

$$= \frac{16}{27}\times10^{-4}(\bar{a}_R 2\cos\theta + \bar{a}_\theta\sin\theta) \text{ (T)}$$

$\bar{B}_e$ (earth) $= -\bar{a}_\theta 10^{-4}$ (T).

Max. deflection when $|B_\theta/B_0|$ is max., or when

$$\left|\frac{B_\theta}{B_0}\right| = \left|\frac{\left(\frac{16}{27}\sin\theta - 1\right)\times10^{-4}}{\frac{16}{27}\times10^{-4}\cos\theta}\right| \text{ is min.}$$

Set $\frac{d}{d\theta}\left(\frac{1-\frac{16}{27}\sin\theta}{\frac{16}{27}\cos\theta}\right) = 0 \longrightarrow \sin\theta = \frac{16}{27}$, or $\theta = 36.34°$.

At $\theta = 36.34°$, $|B_\theta/B_0| = 1.471$ and $\alpha = \tan^{-1}1.471 = 55.8°$.

(If the bar magnet is oriented such that $\bar{B}_m \perp \bar{B}_0$, then $\alpha = 49.8° < 55.8°$.)

P.6-39 $F = \frac{\Phi^2}{\mu_0 S} = \frac{(NI)^2}{\mu_0 S\left(\frac{l_1}{\mu_0 S} + \frac{l_2}{\mu_0 \mu_r S}\right)^2} = \frac{(NI)^2 \mu_0 S}{\left(2l_g + \frac{l_1}{\mu_r}\right)^2}$

$F = 100 \times 9.8 = 980$ (N), $S = 0.01$ (m²), $l_g = 2\times10^{-3}$ (m), $l_1 = 3$ (m), $\mu_r = 4000$.

Solving: $mmf = NI = 1.326\times10^3$ (A-t)

P.6-40 $W_m = \frac{1}{2}\int \mu H^2 dv$

Assume a virtual displacement, Δx, of the iron core.

$$W_m(x+\Delta x) = W_m(x) + \frac{1}{2}\int_{S\Delta x} (\mu-\mu_0) H^2 dv$$
$$= W_m(x) + \frac{1}{2}\mu_0(\mu_r - 1) n^2 I^2 S \Delta x$$

$(F_x)_x = \frac{\partial W_m}{\partial x} = \frac{\mu_0}{2}(\mu_r - 1) n^2 I^2 S$, in the direction of increasing x.

Chapter 7

P.7-1 $\mathcal{V} = -\int_S \frac{\partial \bar{B}}{\partial t}\cdot d\bar{s} = -\int_S \frac{\partial}{\partial t}(\nabla\times\bar{A})\cdot d\bar{s} = -\oint_C \frac{\partial \bar{A}}{\partial t}\cdot d\bar{l}$.

P.7-2 $\bar{B} = \bar{a}_z\, 3\cos(5\pi\times10^7 t - \frac{2}{3}\pi x)\cdot10^{-6}$ (T)

$$\int_S \bar{B}\cdot d\bar{s} = \int_0^{0.2} \bar{a}_z\, 3\cos(5\pi\times10^7 t - \tfrac{2}{3}\pi x)10^{-6}\cdot(\bar{a}_z\, 0.2\, dx)$$
$$= -\frac{0.6}{2\pi}\left[\sin(5\pi\times10^7 t - 0.4\pi) - \sin 5\pi\times10^7 t\right]10^{-6} \quad (Wb)$$

$\mathcal{V} = -\frac{d}{dt}\int \bar{B}\cdot d\bar{s} = 45\left[\cos(5\pi\times10^7 t - 0.4\pi) - \cos 5\pi\times10^7 t\right]$ (V)

$i = \frac{-\mathcal{V}}{2R} = 1.5\left[\cos(5\pi\times10^7 t - 0.4\pi) - \cos 5\pi\times10^7 t\right]$
$= -3\sin(5\pi\times10^7 t - 0.2\pi)\sin(-0.2\pi)$
$= 1.76\sin(5\pi\times10^7 t - 0.2\pi)$ (A)

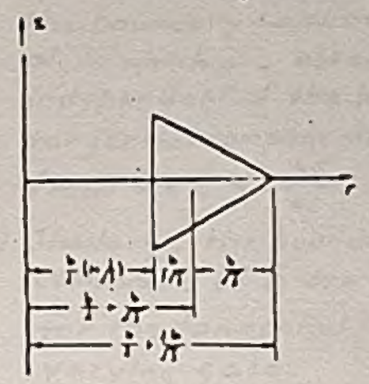
P.7-3 $\bar{B} = \bar{a}_\phi \frac{\mu_0 I \sin\omega t}{2\pi r}$, $\Phi = \int_S \bar{B}\cdot d\bar{s}$, $ds = \bar{a}_\phi\, 2z\, dr$, $z = \frac{\sqrt{3}}{3}(r-d)$

a) $\Phi = \frac{\sqrt{3}}{3}\frac{\mu_0 I \sin\omega t}{\pi}\int_d^{d+b}\left(1 - \frac{d}{r}\right)dr = \frac{\sqrt{3}\mu_0 I\sin\omega t}{3\pi}\left[\frac{\sqrt{3}}{3}b - d\ln\left(\frac{d+b}{d}\right)\right]$

$d = \frac{b}{2}$, $\mathcal{V} = -\frac{d\Phi}{dt} = -\frac{\sqrt{3}\mu_0 I\omega b}{3\pi}\left[\frac{\sqrt{3}}{3} - \frac{1}{2}\ln(\sqrt{3}+1)\right]\cos\omega t$
$= V_m\cos\omega t$

$$V_{rms} = \frac{\sqrt{2}}{2}|V_m| = \frac{\sqrt{2}\mu_0 I\omega b}{12\pi}\left[\sqrt{3} - \ln(\sqrt{3}+1)\right]$$
$$= 0.0472\,\mu_0 I\omega b \quad (V).$$

b)



$$\Sigma = \frac{1}{\sqrt{3}}\left[\frac{b}{2}\left(1+\frac{\sqrt{3}}{3}\right) - r\right].$$

$$\int \bar{B}\cdot d\bar{s} = \frac{\mu_0 I\sin\omega t}{\sqrt{3}\pi}\int_{\frac{1}{2}(1-\frac{\sqrt{3}}{3})}^{\frac{1}{2}(1+\frac{\sqrt{3}}{3})}\left[\frac{b}{2}\left(1+\frac{\sqrt{3}}{3}\right)\frac{1}{r} - 1\right]dr$$

$$= \frac{\mu_0 I\sin\omega t}{\sqrt{3}\pi}\left[\frac{1}{2}\left(1+\frac{\sqrt{3}}{3}\right)\ln\left(\frac{4+\sqrt{3}}{1+\sqrt{3}}\right) - \frac{\sqrt{3}}{3}\right].$$

$$V_{rms} = \frac{1}{\sqrt{2}}\frac{\mu_0 I\omega}{\sqrt{3}\pi}\frac{b}{2}\left|\left(1+\frac{\sqrt{3}}{3}\right)\ln\left(\frac{4+\sqrt{3}}{1+\sqrt{3}}\right) - \sqrt{3}\right|$$
$$= 0.0469\,\mu_0 I\omega b \quad (V).$$

P.7-4 From problem P.6-30: $\Phi_{12} = \mu_0 I(\sin\omega t)\left(d - \sqrt{d^2-b^2}\right)$.

a) $$\mathcal{V} = -\frac{d\Phi}{dt} = -\mu_0 I\omega(\cos\omega t)\left(d - \sqrt{d^2-b^2}\right)$$
$$= V_m\cos\omega t$$

$$3\times10^{-4} = \frac{|V_m|}{\sqrt{2}R} = \frac{\mu_0 I\omega\left(d-\sqrt{d^2-b^2}\right)}{\sqrt{2}R}$$

$$I = \frac{\sqrt{2}R\times3\times10^{-4}}{\mu_0\omega\left(d-\sqrt{d^2-b^2}\right)} = \frac{3\sqrt{2}\times10^{-4}}{4\pi\times10^{-7}(2\pi\times60)\times0.0582} = 0.234\ (A).$$

b) $\alpha = \cos^{-1}\left(\frac{0.2}{0.3}\right) = 48.2°$

P.7-5 $\Phi(t) = B(t)S(t) = (5\cos\omega t)\cdot0.2\,(0.7-x)$
$= 0.35\cos\omega t\,(1+\cos\omega t)$ (mT).

$i = -\frac{1}{R}\frac{d\Phi}{dt} = -\frac{1}{R}0.35\omega(\sin\omega t + \sin 2\omega t)$
$= -1.75\,\omega\sin\omega t\,(1+2\cos\omega t)$ (mA).

P.7-6 $i = \frac{\mathcal{V}}{R} = -\frac{1}{R}\frac{d}{dt}\int_S \bar{B}\cdot d\bar{s} = -\frac{1}{R}\frac{d}{dt}(B_0 hw\cos\omega t)$
$= \frac{\omega B_0 hw}{R}\sin\omega t$.

$P_d = \mathcal{V} i = \frac{(\omega B_0 hw)^2}{R}\sin^2\omega t$ (Power dissipated in R)

On the other hand, for side 1-2: $\bar{F}_{12} = \bar{a}_z\, ihB_0$, $\bar{u}_{12} = -\bar{a}_x\frac{\omega w}{2}\sin\omega t$;
for side 3-4: $\bar{F}_{34} = -\bar{a}_z\, ihB_0$, $\bar{u}_{34} = \bar{a}_x\frac{\omega w}{2}\sin\omega t$.

Mechanical power required to rotate coil: $P_m = -(\bar{F}_{12}\cdot\bar{u}_{12} + \bar{F}_{34}\cdot\bar{u}_{34}) = -(\omega B_0 hwi) = P_d$.

P.7-7 Take the divergence of Eq. (7-37a):

$$\bar{\nabla}\cdot(\bar{\nabla}\times\bar{E}) = -\frac{\partial}{\partial t}(\bar{\nabla}\cdot\bar{B}) = 0 \quad \text{from Eq (2-137)}$$

$\longrightarrow$ $\bar{\nabla}\cdot\bar{B} = f(x,y,z)$, which is to hold at all times everywhere whether $\bar{B}$ exists or not; hence $f(x,y,z)$ must vanish and $\bar{\nabla}\cdot\bar{B}=0$.

Similarly, take the divergence of Eq. (7-37b):

$$\bar{\nabla}\cdot(\bar{\nabla}\times\bar{H}) = \bar{\nabla}\cdot\bar{J} + \frac{\partial}{\partial t}(\bar{\nabla}\cdot\bar{D}) = 0$$

$$= -\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial t}(\bar{\nabla}\cdot\bar{D}), \text{ from Eq. (7-32)}$$

$\longrightarrow$ $\bar{\nabla}\cdot\bar{D} = \rho$.

P.7-8 Eq. (7-46): $\bar{\nabla}\cdot\bar{A} + \mu\epsilon\frac{\partial V}{\partial t} = 0$.

$$\bar{A} = \frac{\mu}{4\pi}\int_{v'}\frac{\bar{J}}{R}dv', \qquad V = \frac{1}{4\pi\epsilon}\int_{v'}\frac{\rho}{R}dv'$$

$$\therefore \quad \frac{\mu_0}{4\pi}\left\{\int_{v'}\left[\bar{\nabla}\cdot\left(\frac{\bar{J}}{R}\right) + \frac{1}{R}\frac{\partial\rho}{\partial t}\right]dv'\right\} = 0. \qquad (i)$$

Now, $\bar{\nabla}\cdot\left(\frac{\bar{J}}{R}\right) = \frac{1}{R}\bar{\nabla}\cdot\bar{J} + \bar{J}\cdot\bar{\nabla}\left(\frac{1}{R}\right) = \bar{J}\cdot\bar{\nabla}\left(\frac{1}{R}\right)$

$$= -\bar{J}\cdot\bar{\nabla}'\left(\frac{1}{R}\right). \qquad (ii)$$

Also, $\bar{\nabla}'\cdot\left(\frac{\bar{J}}{R}\right) = \frac{1}{R}\bar{\nabla}'\cdot\bar{J} + \bar{J}\cdot\bar{\nabla}'\left(\frac{1}{R}\right)$. (iii)

Substituting (iii) in (ii), $\bar{\nabla}\cdot\left(\frac{\bar{J}}{R}\right) = \frac{1}{R}\bar{\nabla}'\cdot\bar{J} - \bar{\nabla}'\cdot\left(\frac{\bar{J}}{R}\right)$. (iv)

Substituting (iv) in (i), $\int_{v'}\frac{1}{R}\left(\bar{\nabla}'\cdot\bar{J} + \frac{\partial\rho}{\partial t}\right)dv' = \int_{v'}\bar{\nabla}'\cdot\left(\frac{\bar{J}}{R}\right)dv'$

Let $R\to\infty$, $S'\to\infty$, $\bar{J}\cdot d\bar{s} = 0$. $= \oint_{S'}\frac{\bar{J}\cdot d\bar{s}'}{R}$.

$\therefore$ $\bar{\nabla}'\cdot\bar{J} + \frac{\partial\rho}{\partial t} = 0$ (Eq. of continuity).

P.7-9 Eq. (7-37b): $\bar{\nabla}\times\bar{H} = \bar{J} + \frac{\partial\bar{D}}{\partial t}$ $\longrightarrow$ $\bar{\nabla}\times\left(\frac{\bar{B}}{\mu}\right) = \bar{J} + \epsilon\frac{\partial\bar{E}}{\partial t}$.

From Eqs. (7-39) and (7-41): $\bar{B} = \bar{\nabla}\times\bar{A}$, $\bar{E} = -\bar{\nabla}V - \frac{\partial\bar{A}}{\partial t}$.

$$\mu\bar{\nabla}\times\left(\frac{1}{\mu}\bar{\nabla}\times\bar{A}\right) = \mu\bar{J} - \mu\epsilon\frac{\partial^2\bar{A}}{\partial t^2} - \mu\epsilon\bar{\nabla}\left(\frac{\partial V}{\partial t}\right).$$

Use gauge condition for potentials in an inhomogeneous medium: $\bar{\nabla}\cdot(\epsilon\bar{A}) + \mu\epsilon^2\frac{\partial V}{\partial t} = 0$

$\longrightarrow$ Wave equation for vector potential:

$$-\mu\bar{\nabla}\times\left(\frac{1}{\mu}\bar{\nabla}\times\bar{A}\right) + \mu\epsilon\bar{\nabla}\left[\frac{1}{\mu\epsilon^2}\bar{\nabla}\cdot(\epsilon\bar{A})\right] - \mu\epsilon\frac{\partial^2\bar{A}}{\partial t^2} = -\mu\bar{J}.$$

From Eq. (7-37c), $\bar{\nabla}\cdot\bar{D} = \rho$ $\longrightarrow$ $\bar{\nabla}\cdot(\epsilon\bar{\nabla}V) + \frac{\partial}{\partial t}\bar{\nabla}\cdot(\epsilon\bar{A}) = -\rho$.

Wave equation for scalar potential: $\frac{1}{\epsilon}\bar{\nabla}\cdot(\epsilon\bar{\nabla}V) - \mu\epsilon\frac{\partial^2 V}{\partial t^2} = -\rho$.

P.7-12 As shown in problem P.7-7:

a) Eq. (7-37d) can be derived from Eq. (7-37a). Hence, the boundary conditions for the normal components of $\bar{B}$, which are obtained from $\bar{\nabla}\cdot\bar{B}=0$, are not independent of the boundary conditions for the tangential components of $\bar{E}$, which are obtained from $\bar{\nabla}\times\bar{E} = -\frac{\partial\bar{B}}{\partial t}$.

b) Similarly, the boundary conditions for the normal components of $\bar{D}$ are not independent of those for the tangential components of $\bar{H}$ in the time-varying case.

P.7-13 Medium 1: Free space; medium 2: $\mu_2\to\infty$.

$\bar{H}_2$ must be zero so that $\bar{B}_2$ is not infinite.

$$\therefore \quad \bar{a}_{n2}\times\bar{H}_1 = 0 \;; \quad B_{n1} = B_{n2}$$

$$\bar{a}_{n2}\times(\bar{D}_1 - \bar{D}_2) = \rho_s \;; \quad E_{t1} = E_{t2}.$$

P.7-14 $\bar{E}_1(z,t) = \bar{a}_x 0.03\sin 10^8\pi(t-\frac{z}{c}) = \bar{a}_x R_e\left[0.03\, e^{-j\pi z}e^{j10^8\pi(t-z/c)}\right]$

$\bar{E}_2(z,t) = \bar{a}_x 0.04\cos\left[10^8\pi(t-\frac{z}{c}) - \frac{\pi}{3}\right] = R_e\left[0.04\, e^{-j\pi z}e^{j10^8\pi(t-z/c)}\right]$.

Phasors: $\bar{E} = \bar{E}_1 + \bar{E}_2 = \bar{a}_x\left[0.03\, e^{-j\pi z} + 0.04\, e^{-j\pi z}\right]$

$$= \bar{a}_x\left[-j0.03 + (0.02 - j0.02\sqrt{3})\right]$$

$$= \bar{a}_x(0.068\, e^{-j1.27}) = \bar{a}_x E_0 e^{-j\theta}.$$

$\therefore$ $E_0 = 0.068$, $\theta = -1.27$ rad. or $-73.8°$.

P.7-16 $\bar{\nabla}^2 V - \mu\epsilon\frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\epsilon}$ with $V(R,t) = \frac{1}{4\pi\epsilon}\int_{v'}\frac{\rho(t-R/u)}{R}dv'$

$\bar{\nabla}^2 V - \frac{1}{u^2}\frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\epsilon}$.

We need $\bar{\nabla}'\left(\frac{\rho}{R}\right) = \frac{1}{R}\bar{\nabla}'\rho + \rho\bar{\nabla}'\left(\frac{1}{R}\right) + 2(\bar{\nabla}\rho)\cdot\bar{\nabla}\left(\frac{1}{R}\right)$.

{Formula: $\bar{\nabla}^2(fg) = \bar{\nabla}\cdot\bar{\nabla}(fg) = g\bar{\nabla}^2 f + f\bar{\nabla}^2 g + 2(\bar{\nabla}f)\cdot(\bar{\nabla}g)$}

Let $\zeta = t - \sqrt{\mu\epsilon}R = t - R/u$, $\rho(\zeta) = \rho(t-R/u)$

$\bar{\nabla}^2\rho(\zeta) = \frac{1}{u^2}\frac{d^2\rho}{d\zeta^2} - \frac{2}{uR}\frac{d\rho}{d\zeta}$, $\bar{\nabla}^2\left(\frac{1}{R}\right) = -4\pi\delta(R)$.

$(\bar\nabla\rho)\cdot(\bar\nabla\frac{1}{R}) = \frac{\partial\rho(r)}{\partial R}\left(-\frac{1}{R^2}\right) - \frac{1}{uR^2}\frac{d\rho}{d\zeta}$.

Substituting back, $\bar\nabla^2\left(\frac{\rho}{R}\right) = \frac{1}{u^2R}\frac{d^2\rho}{d\zeta^2} - 4\pi\rho\delta(R)$.

$\bar\nabla^2 V = \frac{1}{4\pi\epsilon}\bar\nabla^2\int_{v'}\frac{\rho}{R}dv' = \frac{1}{4\pi\epsilon}\int_{v'}\left[\frac{1}{u^2R}\frac{d^2\rho}{d\zeta^2} - 4\pi\rho\delta(R)\right]dv'$

$\frac{\partial^2 V}{\partial t^2} = \frac{1}{4\pi\epsilon}\int_{v'}\frac{1}{R}\frac{d^2\rho}{d\zeta^2}dv'$

$\therefore \bar\nabla^2 V - \frac{1}{u^2}\frac{\partial^2 V}{\partial t^2} = \frac{1}{4\pi\epsilon}\int\left[\frac{1}{u^2R}\frac{d^2\rho}{d\zeta^2} - 4\pi\rho\delta(R) - \frac{1}{u^2R}\frac{d^2\rho}{d\zeta^2}\right]dv'$

$= -\frac{\rho(R)}{\epsilon}$. Q.E.D.

P.7-17 $\bar\nabla\times\bar E = -\mu\frac{\partial\bar H}{\partial t}$ ① $\qquad \bar\nabla\times\bar H = \bar J + \epsilon\frac{\partial\bar E}{\partial t}$ ②

$\bar\nabla\cdot\bar E = \frac{\rho}{\epsilon}$ ③ $\qquad \bar\nabla\cdot\bar H = 0$ ④

$\bar\nabla\times$ ①: $\bar\nabla\times\bar\nabla\times\bar E = -\mu\frac{\partial}{\partial t}\left(\bar J + \epsilon\frac{\partial\bar E}{\partial t}\right)$.

$\bar\nabla(\bar\nabla\cdot\bar E) - \nabla^2\bar E = -\mu\frac{\partial\bar J}{\partial t} + \mu\epsilon\frac{\partial^2\bar E}{\partial t^2}$.

Wave equation for $\bar E$: $\nabla^2\bar E - \mu\epsilon\frac{\partial^2\bar E}{\partial t^2} = \mu\frac{\partial\bar J}{\partial t} + \frac{1}{\epsilon}\bar\nabla\rho$.

$\bar\nabla\times$ ②: $\bar\nabla\times\bar\nabla\times\bar H = \bar\nabla\times\bar J + \epsilon\frac{\partial}{\partial t}(\bar\nabla\times\bar E)$.

$\bar\nabla(\bar\nabla\cdot\bar H) - \nabla^2\bar H = \bar\nabla\times\bar J - \mu\epsilon\frac{\partial^2\bar H}{\partial t^2}$.

Wave equation for $\bar H$: $\nabla^2\bar H - \mu\epsilon\frac{\partial^2\bar H}{\partial t^2} = -\bar\nabla\times\bar J$.

For sinusoidal time dependence: $\frac{\partial}{\partial t}\to j\omega$, $\frac{\partial^2}{\partial t^2}\to(j\omega)^2$.

Helmholtz's equations (for phasors): $\nabla^2\bar E + \omega^2\mu\epsilon\bar E = j\omega\mu\bar J + \frac{1}{\epsilon}\bar\nabla\rho$

$\nabla^2\bar H + \omega^2\mu\epsilon\bar H = -\bar\nabla\times\bar J$.

P.7-18 $\bar E = \bar a_y\, 0.1\sin(10\pi x)\cos(6\pi10^8 t - \beta z)$ (V/m)

Use phasors:

$\bar H = -\frac{1}{j\omega\mu_0}\bar\nabla\times\bar E = -\frac{1}{j\omega\mu_0}\begin{vmatrix}\bar a_x & \bar a_y & \bar a_z\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ 0 & E_y & 0\end{vmatrix}$

$= \frac{1}{\omega\mu_0}\left[\bar a_x\, j0.1\beta\sin(10\pi x) + \bar a_z\, 0.1(10\pi)\cos(10\pi x)\right]e^{-j\beta z}$ ①

$\bar E = \frac{1}{j\omega\epsilon_0}\nabla\times\bar H = \bar a_y\frac{0.1}{\omega^2\mu_0\epsilon_0}\left[(10\pi)^2+\beta^2\right]\sin(10\pi x)\,e^{-j\beta z}$ ①

Phasor form for given $\bar E$:

$\bar E = \bar a_y\, 0.1\sin(10\pi x)\,e^{-j\beta z}$. ②

Equating ② and ①: $(10\pi)^2 + \beta^2 = \omega^2\mu_0\epsilon_0 = 400\pi^2$.

$\therefore \beta = \sqrt{300}\,\pi = 10\sqrt{3}\,\pi = 54.4$ (rad/m).

From ①, $\bar H(x,z;t) = Re(H e^{j\omega t})$

$= -\bar a_x\, 2.30\times10^{-4}\sin(10\pi x)\cos(6\pi10^8 t - 54.4z)$

$-\bar a_z\, 1.33\times10^{-4}\cos(10\pi x)\sin(6\pi10^8 t - 54.4z)$ (A/m)

P.7-19 $\bar H(x,z;t) = \bar a_y\, 2\cos(15\pi x)\sin(6\pi10^8 t - \beta z)$ (A/m).

Phasor: $\bar H = \bar a_y\, 2\cos(15\pi x)\,e^{-j\beta z}$

$\beta^2 + (15\pi)^2 = \omega^2\mu_0\epsilon_0 = (6\pi10^8)^2\frac{1}{(3\times10^8)^2}$

$\beta^2 = 400\pi^2 - 225\pi^2 = 175\pi^2$

$\beta = 13.2\pi = 41.6$ (rad/m).

$\bar E = \frac{1}{j\omega\epsilon_0}\bar\nabla\times\bar H = \frac{1}{j\omega\epsilon_0}\left(-\bar a_x\frac{\partial H_z}{\partial x} + \bar a_z\frac{\partial H_x}{\partial x}\right)$

$= -j6\left[-\bar a_x\, j2\beta\cos(15\pi x) - \bar a_z\, 30\pi\sin(15\pi x)\right]e^{-j\beta z}$

$= \left[-\bar a_x\, 158\pi\cos(15\pi x) + \bar a_z\, j\,150\pi\sin(15\pi x)\right]e^{-j\beta z}$

$\bar E(x,z;t) = Im(\bar E e^{j\omega t})$

$= \bar a_x\, 496\cos(15\pi x)\sin(6\pi10^8 t - 41.6z)$

$+ \bar a_z\, 565\sin(15\pi x)\cos(6\pi10^8 t - 41.6z)$ (V/m).

P.7-20 $\bar E = \bar a_\theta\frac{E_0}{R}\sin\theta\cos(\omega t - kR)$. ①

$\bar\nabla\times\bar E = \bar a_\phi\frac{1}{R}\frac{\partial}{\partial R}(RE_\theta) = \bar a_\phi\frac{E_0 k}{R}\sin\theta\sin(\omega t - kR)$

$= -\mu_0\frac{\partial H}{\partial t}$ ⟶ $\bar H = \bar a_\phi\int\frac{E_0 k}{\mu_0 R}\sin\theta\sin(\omega t - kR)\,dt$

$= \bar a_\phi\frac{E_0 k}{\omega\mu_0 R}\sin\theta\cos(\omega t - kR)$. ②

Also, $\bar\nabla\times\bar H = \epsilon_0\frac{\partial\bar E}{\partial t}$ ⟶ $\frac{\partial\bar E}{\partial t} = \frac{1}{\epsilon_0}\bar\nabla\times\bar H = -\bar a_\theta\frac{1}{R}\frac{\partial}{\partial R}(RH_\phi)$

$\bar E = \bar a_\theta\int\frac{E_0 k^2}{\omega\mu_0\epsilon_0 R}\sin\theta\sin(\omega t - kR)\,dt$

Comparing ① and ②, $k = \omega\sqrt{\mu_0\epsilon_0}$. $\quad = \bar a_\theta\frac{E_0 k^2}{\omega^2\mu_0\epsilon_0 R}\sin\theta\sin(\omega t - kR)$. ③

From ②, $\bar H = \bar a_\phi\frac{E_0}{R}\sqrt{\frac{\epsilon_0}{\mu_0}}\sin\theta\cos\omega(t - \sqrt{\mu_0\epsilon_0}R)$.

P7-21 Maxwell's curl equations: $\bar{\nabla}\times\bar{E} = -j\omega\mu\bar{H}$ ①

$\bar{\nabla}\times\bar{H} = j\omega\epsilon\bar{E}$ ②

From $\bar{\nabla}\cdot\bar{E}=0$, define $\bar{A}_e$ such that $\bar{E}=\bar{\nabla}\times\bar{A}_e$ ③

From ①, $\bar{H} = \frac{j}{\omega\mu}\bar{\nabla}\times\bar{E} = \frac{j}{\omega\mu}\bar{\nabla}\times\bar{\nabla}\times\bar{A}_e$

$= \frac{j}{\omega\mu}\left[\bar{\nabla}(\bar{\nabla}\cdot\bar{A}_e) - \bar{\nabla}^2\bar{A}_e\right]$. ④

From ②, $\bar{\nabla}\times(\bar{H}-j\omega\epsilon\bar{A}_e)=0$, let $\bar{H}-j\omega\epsilon\bar{A}_e = -\bar{\nabla}V_m$. ⑤

Subtracting ⑤ from ④: $\omega\epsilon\bar{A}_e = \frac{1}{\omega\mu}\left[\bar{\nabla}(\bar{\nabla}\cdot\bar{A}_e)-\bar{\nabla}^2\bar{A}_e\right] - j\bar{\nabla}V_m$. ⑥

Choose $\bar{\nabla}\cdot\bar{A}_e = -j\omega\mu V_m$.

a) Eq. ⑤ becomes $\bar{H} = j\omega\epsilon\bar{A}_e + \frac{j}{\omega\mu}\bar{\nabla}(\bar{\nabla}\cdot\bar{A}_e)$.

b) Eq. ⑥ becomes $\bar{\nabla}^2\bar{A}_e + \omega^2\mu\epsilon\bar{A}_e = 0$, a homogeneous Helmholtz eq.

P7-22 $\bar{H} = j\omega\epsilon_0\bar{\nabla}\times\bar{\Pi}_e$ ①

$\bar{\nabla}\times\bar{E} = -j\omega\mu_0\bar{H} = \omega^2\mu_0\epsilon_0\bar{\nabla}\times\bar{\Pi}_e$

$\bar{\nabla}\times(\bar{E}-k_0^2\bar{\Pi}_e)=0 \longrightarrow$ Let $\bar{E}-k_0^2\bar{\Pi}_e = \bar{\nabla}V_e$ ②

$\bar{\nabla}\times\bar{H} = j\omega\bar{D} - j\omega(\epsilon_0\bar{E}+\bar{P}) = j\omega\epsilon_0(\bar{E}+\frac{\bar{P}}{\epsilon_0})$. ③

Substituting ① and ② in ③:

$j\omega\epsilon_0\bar{\nabla}\times\bar{\nabla}\times\bar{\Pi}_e - j\omega\epsilon_0(k_0^2\bar{\Pi}_e + \bar{\nabla}V_e + \frac{\bar{P}}{\epsilon_0})$

$= -j\omega\epsilon_0(\bar{\nabla}\bar{\nabla}\cdot\bar{\Pi}_e - \bar{\nabla}^2\bar{\Pi}_e)$. ④

Choose $\bar{\nabla}\cdot\bar{\Pi}_e = V_e$. Eq. ④ becomes

b) $\bar{\nabla}^2\bar{\Pi}_e + k_0^2\bar{\Pi}_e = -\frac{\bar{P}}{\epsilon_0}$; (7-95)

a) Eq. ① becomes

$\bar{E} = k_0^2\bar{\Pi}_e + \bar{\nabla}\bar{\nabla}\cdot\bar{\Pi}_e$

$= k_0^2\bar{\Pi}_e + (\bar{\nabla}^2\bar{\Pi}_e + \bar{\nabla}\times\bar{\nabla}\times\bar{\Pi}_e)$. ⑤

Combination of Eqs (7-95) and ① gives

$\bar{E} = \bar{\nabla}\times\bar{\nabla}\times\bar{\Pi}_e - \frac{\bar{P}}{\epsilon_0}$.

P7-23 a) $\left|\frac{\text{Displacement current}}{\text{Conduction current}}\right| = \frac{\omega\epsilon}{\sigma} = \frac{(2\pi\times100\times10^6)\times\frac{1}{4\pi}\times10^{-9}}{5.70\times10^7} = 9.75\times10^{-4}$

b) $\bar{\nabla}\times\bar{H}=\sigma\bar{E}$, $\bar{\nabla}\times\bar{E} = -j\omega\mu\bar{H}$

$\bar{\nabla}\times\bar{\nabla}\times\bar{H} = \bar{\nabla}(\bar{\nabla}\cdot\bar{H}) - \bar{\nabla}^2\bar{H} = \sigma\bar{\nabla}\times\bar{E}$.

But $\bar{\nabla}\cdot\bar{H}=0$.

$\therefore\ \bar{\nabla}^2\bar{H} + \sigma\bar{\nabla}\times\bar{E} = 0$.

or, $\bar{\nabla}^2\bar{H} - j\omega\mu\sigma\bar{H} = 0$.

Chapter 8

P8-2 Harmonic time dependence: $e^{j\omega t}$; $\frac{\partial}{\partial t}\to j\omega$

Let phasors $\bar{E} = \bar{E}_0 e^{-j\bar{k}\cdot\bar{x}}$

$\bar{H} = \bar{H}_0 e^{-j\bar{k}\cdot\bar{x}}$,

where $\bar{E}_0$ and $\bar{H}_0$ are constant vectors.

Maxwell's equations: $\bar{\nabla}\times\bar{E} = \bar{\nabla}(e^{-j\bar{k}\cdot\bar{x}})\times\bar{E}_0 = -j\omega\mu\bar{H}$

$\bar{\nabla}\times\bar{H} = \bar{\nabla}(e^{-j\bar{k}\cdot\bar{x}})\times\bar{H}_0 = j\omega\epsilon\bar{E}$

$\bar{\nabla}\cdot\bar{E} = \bar{\nabla}(e^{-j\bar{k}\cdot\bar{x}})\cdot\bar{E}_0 = 0$

$\bar{\nabla}\cdot\bar{H} = \bar{\nabla}(e^{-j\bar{k}\cdot\bar{x}})\cdot\bar{H}_0 = 0$.

But $\bar{\nabla}(e^{-j\bar{k}\cdot\bar{x}}) = e^{-j\bar{k}\cdot\bar{x}}\bar{\nabla}(-j\bar{k}\cdot\bar{x}) = e^{-j\bar{k}\cdot\bar{x}}\left[-j\bar{\nabla}(k_x x + k_y y + k_z z)\right]$

$= -j(\bar{a}_x k_x + \bar{a}_y k_y + \bar{a}_z k_z)e^{-j\bar{k}\cdot\bar{x}} = -j\bar{k}e^{-j\bar{k}\cdot\bar{x}}$

$\therefore$ Maxwell's equations become: $\bar{k}\times\bar{E} = \omega\mu\bar{H}$, $\bar{k}\times\bar{H} = -\omega\epsilon\bar{E}$

$\bar{k}\cdot\bar{E}=0$, $\bar{k}\cdot\bar{H}=0$.

P8-3 $\bar{H} = \bar{a}_z\, 4\times10^{-4}\cos(10^7\pi t - k_0 y + \frac{\pi}{4})$ (A/m).

a) $k_0 = \omega\sqrt{\mu_0\epsilon_0} = \frac{10^7\pi}{3\times10^8} = \frac{\pi}{30} = 0.105$ (rad/m).

At $t = 3\times10^{-9}$ (s), we require

$10^7\pi(3\times10^{-9}) - \frac{\pi}{30}y + \frac{\pi}{4} = \pm n\pi + \frac{\pi}{2}$; $n=0,1,2,\cdots$

$y = \pm 30n - 7.5$ (m).

But $\lambda = \frac{2\pi}{k_0} = 60$ (m), $\therefore\ y = 22.5 \pm n\lambda/2$ (m).

b) $\bar{E} = \eta_0\bar{H}\times\bar{a}_y = -\bar{a}_x\, 1.508\times10^{-1}\cos(10^7\pi t - \frac{\pi}{30}y + \frac{\pi}{4})$ (V/m).

P8-4 Let $\alpha = \omega t - kz$, $\bar{E} = \bar{a}_x E_{1e}\sin\alpha + \bar{a}_y E_{2e}\sin(\alpha+\psi)$

$= \bar{a}_x E_x + \bar{a}_y E_y$.

$\frac{E_x}{E_{1e}} = \sin\alpha$, $\frac{E_y}{E_{2e}} = \sin(\alpha+\psi) = \sin\alpha\cos\psi + \cos\alpha\sin\psi$

$= \frac{E_x}{E_{1e}}\cos\psi + \sqrt{1-\left(\frac{E_x}{E_{1e}}\right)^2}\sin\psi$.